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A Fuzzy Programming Approach for solving a p-Center Problem under Uncertainty

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A R T I C L E I N F O

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A B S T R A C T

Facility location problems have often vagueness and uncertain properties. In P-center problems, this uncertainty can be in the parameters of demand nods. Firstly, in this paper, a vertex-center problem with uncertain demand nodes is considered in which the demand nodes are fuzzy and fuzzy random variables. Then, new solving methods are proposed based on possibility and necessity measures, using fuzzy and fuzzy random programming, respectively. Finally, a real case study in the city of Tabriz in Iran is presented to clarify the methods discussed in this paper. The computational results of the study indicate that these methods can be implemented for center problem with uncertain framework.

1. Introduction

Facility location is an issue related to the planning stage of a factory or a service unit. Decision-making regarding facility locations is a strategic matter. Constructing a new facility is usually costly, and the impact of this decision will last for a long time. In the context of location, facilities are placed in an environment that changes over time; consequently, in a cost location model, demand, travel time, and other inputs related to the model can be highly uncertain. This has led to the development of models for facility location under conditions of uncertainty being a high priority(Bagherinejad & Shoeib, 2018; Cheng, Adulyasak, & Rousseau, 2021).

Location problems are typically classified based on the type of objective function, parameters, solutions, etc. These problems vary in terms of objective functions, such as P-

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Center, P-median, and covering problems, among others. If the objective function is defined as minimizing the maximum weighted distance between points and facilities, the problem is classified as a P-center problem. If the facility locations is limited to the points of the network, the problem is classified as a vertex P-center problem; however, if the facility can be placed anywhere in the network, it becomes a free P-center problem (Contardo, Iori, & Kramer, 2019; Silva et al., 2021).

The P-center problem was first formulated as the vertex P-center problem. This problem involves selecting P-centers from a limited set of nodes and assigning a set of clients to them, with the goal of minimizing the maximum difference between a client and its associated center (Nematian & Musavi, 2016).The vertex P-center model, which is presented with weighted demand distances (Nematian & Sadati, 2015), serves as the foundational model used in this study.

Considering that uncertainty in information can completely undermine the validity of an optimal solution obtained, incorporating this uncertainty into achieving an optimal solution is of great importance. (Taghavi and Shavandi 2012) employed certain non-deterministic optimization methods to examine the P-center problem under demand uncertainty and developed its non-deterministic model as an integer programming model, solving it using a tabu search algorithm. (Duran-Mateluna et al. 2023) presented a robust integer programming model for the weighted vertex P-center problem in such a way that the weights of the nodes and the lengths of the edges were uncertain.

Another method for considering parameter uncertainty is the use of fuzzy logic and possibility theory. In this approach, inaccuracies and uncertainties in the model can be incorporated using the opinions of experts and decision-makers in modeling the problem. In this approach, we aim to provide a new modeling framework for the P-center problem under fuzzy uncertainty using possibility theory. This employs integer programming with fuzzy parameters, and through the fuzzy CCP approach based on possibility theory, the resulting models will be transformed into deterministic forms. To achieve this, the approach presented by (Nematian 2015a) will be utilized.

Additionally, considering that in real-world problems, parameter values possess not only fuzzy properties but also random properties, it is necessary for experts and decision-makers to make decisions based on information that has both fuzzy ambiguity and probabilistic uncertainty. For this purpose, integer programming models with Fuzzy Random Linear Programming (FRLP) are utilized. This is a Fuzzy Random Linear Programming (FRLP) where the parameters are random fuzzy variables, while the decision variables are real numbers (Nematian, 2015b; Wang, Feng, and Fei, 2024). Subsequently, efforts will be made to transform the obtained models, using the principle of fuzzy extension and the presented possibility theory, into deterministic linear programming problems. The general method for solving the problem is derived from a technique proposed by (Nematian, 2015a; Nematian, 2015b).

This paper is composed of the following sections: Section 2 addresses the modeling and solving of the P-center problem under uncertainty using fuzzy linear programming and random fuzzy linear programming. Section 3 examines a case study of family park location within the city of Tabriz to assess the effectiveness of the methods presented in this paper. Finally, Section 4 includes the final conclusion and discussion for future research.

2. Parameters and Variables of the Model

The parameters and variables of the above model are defined as follows:

- d_{ij}: The shortest path length between demand node *i* and candidate facility node *j*.
- w_i: Represents the demand at each node *i*.

- p: The number of facilities to be located.
- x_i: A binary variable indicating if a facility is located at node j.
- x_{ij}: A binary variable indicating the assignment of node i to the facility located at node j.
- T: The maximum distance between a demand node and the nearest facility.

2.1. Modeling the P-Center Problem under Fuzzy Uncertainty

In this study, the foundational model of the vertex P-center problem with weighted distances under fuzzy uncertainty is examined as follows (Nematian & Musavi, 2016):

Model: 1

Min T (1)

$$\sum_{j=1}^{n} w_i X_{ij} d_{ij} \leq T \quad \forall i \quad (2)$$

$$\sum_{j=1}^{n} X_j = P \quad (3)$$

$$X_{ij} - X_j \leq 0 \quad \forall i,j \quad (4)$$

$$\sum_{j=1}^{n} X_{ij} = 1 \quad \forall i \quad (5)$$

$$X_{ij} \in \{0,1\} \quad \forall i,j \quad (6)$$

$$X_j \in \{0,1\} \quad \forall i \quad (7)$$

In explaining the above model, it can be stated that the objective function, along with constraint (1), is aimed at minimizing the maximum weighted distance, which is the essence of the P-center problem. This primary objective function can be expressed as **follows:**

$$Min\ \{ Max_i \sum_{j=1}^{n} d_{ij} X_{ij} \} \quad (8)$$

Constraint (2) ensures that all demands can only be satisfied if the facilities are located at the designated nodes i. Constraint (3) states that demand at node i can only be assigned to a facility that has been established at node j. Constraint (4) indicates that the demand of each node must be met by a facility that has been previously established.

2.2. Fuzzy Linear Programming (FLP)

Fuzzy linear programming (FLP), w_i is considered in the form of triangular fuzzy numbers. Therefore, we have:

Model 2

Min Z (9)

$$\sum_{j=1}^{n} \widetilde{w}_i X_{ij} d_{ij} \leq Z \quad \forall i \quad (10)$$

Constraints 3-7 from Model 1

2.3. Model Based on Possibility Criteria in Fuzzy Linear Programming

If the permissible level of possibility (CCP) is considered, Model 2 is approached with the parameter η:

Min Z (11)

$$\prod\left(\sum_{j=1}^{n} \widetilde{w}_i X_{ij} d_{ij} \leq Z\right) \geq \tau \qquad \forall i \qquad (12)$$

Constraints 3-7 from Model 1

Theorem 1

If the decision vector is non-negative, we will have the following relation:

$$\prod\left(\sum_{j=1}^{n} \widetilde{w}_i X_{ij} d_{ij} \leq Z\right) \geq \tau \leftrightarrow \sum_{j=1}^{n} w_i X_{ij} d_{ij} - L(\tau) \sum_{j=1}^{n} \beta_i X_{ij} d_{ij} \leq Z \qquad (13)$$

To prove the theorems, you can refer to Nematian (2015a, 2015b) where $L*(\lambda)=\sup\{t|L(t)\geq\lambda\}$, with L representing the inverse function of L^* in the above relation. Based on the result obtained from the theorem 1, model 3 is as follows:

Model 3

$$\min \ Z \qquad (14)$$

$s.t.$

$$\sum_{j=1}^{n} w_i^0 d_{ij} x_{ij} - L^*(\eta) \sum_{j=1}^{n} \beta_i d_{ij} x_{ij} \leq Z, \quad \forall i, \qquad (15)$$

$$\sum_{j=1}^{n} X_j = P, \qquad (16)$$

$$X_{ij} - X_j \leq 0, \qquad \forall\, i, j, \qquad (17)$$

$$\sum_{j=1}^{n} X_{ij} = 1, \qquad \forall\ i, \qquad (18)$$

$$X_{ij} \in \{0, 1\}, \qquad \forall\, i, j, \qquad (19)$$

$$X_j \in \{0, 1\}, \qquad \forall\, j. \qquad (20)$$

This model is formulated as a deterministic linear programming model and can be easily solved using linear programming software.

2.4. Model Based on Necessity Criteria in Fuzzy Linear Programming

Since the solution obtained from the model based on possibility criteria is quite optimistic, this model is not suitable for pessimistic decision-makers who intend to avoid risk. Therefore, another type of model is developed, referred to as the model based on necessity criteria, which is appropriate for pessimistic individuals and those who are risk-averse.

Model 4

Min Z

(21)

$$N(\sum_{j=1}^{n} \widetilde{w}_i X_{ij} d_{ij} \leq Z) \geq \tau \qquad \forall i \tag{22}$$

Theorem 2: If the decision vector *x* is non-negative, relation (22) changes as follows:

$$N\left(\sum_{j=1}^{n} \widetilde{w}_i X_{ij} d_{ij} \leq Z\right) \geq \tau \leftrightarrow \sum_{j=1}^{n} w_i X_{ij} d_{ij} - L^{*}(1-\tau) \sum_{j=1}^{n} \beta_i X_{ij} d_{ij} \leq Z \tag{23}$$

Therefore, model 5 changes as follows:

Model 5

$$\min \ Z \tag{24}$$

$$s.t. \sum_{j=1}^{n} w_i^0 \, d_{ij} x_{ij} - L^{*}(1-\eta) \sum_{j=1}^{n} \beta_i \, d_{ij} x_{ij} \leq Z, \qquad \forall i, \tag{25}$$

$$\sum_{j=1}^{n} X_j = P, \tag{26}$$

$$X_{ij} - X_j \leq 0, \qquad \forall \, i, j, \tag{27}$$

$$\sum_{j=1}^{n} X_{ij} = 1, \qquad \forall \, i, \tag{28}$$

$$X_{ij} \in \{0, 1\}, \qquad \forall \, i, j, \tag{29}$$

$$X_j \in \{0, 1\}, \qquad \forall \, j. \tag{30}$$

2.5. Fuzzy Random Variable (FRV) Programming

Considering that the uncertainty in the parameter *wi* has both fuzzy and random properties, it seems appropriate to present a model based on random fuzzy linear programming for this problem. This model is built on the concept of Fuzzy Random Variables (FRV), where an FRV is a random variable whose actual value is a fuzzy number. Therefore, this concept encompasses both aspects of possibility and probability, simultaneously incorporating fuzzy and random properties.

The FRLP model for the vertex P-center problem with uncertain weighted distances and coefficients of fuzzy random variables is presented as follows:

Model 6:

$$\min \ Z \tag{31}$$

$$s.t. \ \sum_{j=1}^{n} \tilde{w}_i x_{ij} d_{ij} \le Z, \quad \forall i, \tag{32}$$

$$\sum_{j=1}^{n} X_j = P, \tag{33}$$

$$X_{ij} - X_j \le 0, \qquad \forall \, i, j, \tag{34}$$

$$\sum_{j=1}^{n} X_{ij} = 1 \,, \qquad \forall \ i, \tag{35}$$

$$X_{ij} \in \{0,1\}, \qquad \forall \, i, j, \tag{36}$$

$$X_j \in \{0,1\} \,, \qquad \forall \, j \,. \tag{37}$$

2.6. Model Based on Possibility Criteria in Fuzzy Random Variable Programming

In order to control fuzzy random variables, the programming model of the problem is defined as follows, using possibility theory:

Model 7

$$\min \ Z \tag{38}$$

$$s.t. \ \Pr\left\{\omega \middle| \prod\left(\sum_{j=1}^{n} \tilde{w}_i(\omega) x_{ij} d_{ij} \le Z \right) \ge \eta \right\} \ge \theta, \qquad \forall i, \tag{39}$$

Constraints 33-37 from model 6.

In the above model, η is the permissible possibility level, and θ is the permissible probability level.

Theorem 3: If the decision vector x is non-negative, the following two relations are equivalent:

$$\Pr\left\{\omega \middle| \prod\left(\sum_{j=1}^{n} \tilde{w}_i(\omega) x_{ij} d_{ij} \le Z \right) \ge \eta \ \right\} \ge \theta \Leftrightarrow \tag{40}$$

$$T^*(\theta) \sum_{j=1}^{n} w_i^{(2)} d_{ij} x_{ij} + \sum_{j=1}^{n} w_i^{(0)} d_{ij} x_{ij} - L^*(\eta) \sum_{j=1}^{n} \beta_i d_{ij} x_{ij} \le Z,$$

It is defined as $T*(\lambda)=\inf\{t|T(t)\ge\lambda\}$, and $T*$ is the inverse function of T. Based on the relation obtained in Theorem 3, Model 7 is transformed as follows:

Model 8

$$\min \ Z \tag{41}$$

$$s.t. \quad \sum_{j=1}^{n} T^*(\theta) w_i^{(2)} d_{ij} x_{ij} + \sum_{j=1}^{n} w_i^{(0)} d_{ij} x_{ij} - L^*(\eta) \sum_{j=1}^{n} \beta_i d_{ij} x_{ij} \le Z, \quad \forall i, \tag{42}$$

Constraints 33-37 from model 6.

2.7. Model Based on Necessity Criteria in Fuzzy Random Variable Programming

With FRLP variables for the vertex P-center problem with weighted distances in the -P state, this model, as previously mentioned, pertains to pessimistic individuals and is presented for the fuzzy random problem as follows:

Model 9

$$\min \ Z \tag{43}$$

$$s.t. \quad \Pr\{\omega \mid \mathrm{N}(\textstyle\sum_{j=1}^{n} \tilde{w}_i(\omega) x_{ij} d_{ij} \le Z) \ge \eta \ \} \ge \theta, \quad \forall i, \tag{44}$$

Constraints 33-37 from model 6.

Theorem 4: If the decision vector *x* is non-negative, we will have:

$$\Pr\{\omega \mid \mathrm{N}(\textstyle\sum_{j=1}^{n} \tilde{w}_i(\omega) x_{ij} d_{ij} \le Z) \ge \eta \ \} \ge \theta \Leftrightarrow$$

$$T^*(\theta) \sum_{j=1}^{n} w_i^{(2)} d_{ij} x_{ij} + \sum_{j=1}^{n} w_i^{(0)} d_{ij} x_{ij} - L^*(1-\eta) \sum_{j=1}^{n} \beta_i d_{ij} x_{ij} \le Z . \tag{45}$$

And the final model based on necessity criteria is presented as follows:

Model 10

$$\min \ Z \tag{46}$$

$$s.t. \quad \sum_{j=1}^{n} w_i^{(0)} d_{ij} x_{ij} + T^*(\theta) \sum_{j=1}^{n} w_i^{(2)} d_{ij} x_{ij} - L^*(1-\eta) \sum_{j=1}^{n} \beta_i d_{ij} x_{ij} \le Z, \quad \forall i, \tag{47}$$

Constraints 33-37 from model 6.

3. Case Study in the City of Tabriz

In this section, a case study has been conducted in the city of Tabriz regarding the location and allocation of family parks. The objective is to locate family parks in such a way that the maximum weighted distance in the network is minimized. Therefore, the problem is of type -P. To conduct the study, the map of Tabriz city and candidate points for locating family parks are shown in Table 1.

Table 1

Names and Numbers of Regions

Region Name	Number	Region Name	Number	Region Name	Number
Farhang	1	Khayyam	5	Koye Enghelab	9
Bahar Street	2	17th Shahrivar	6	Zaferaniyeh	10
Sheshgolan	3	Ashrafi Laleh	7	Vali Asr	11
Abbasi	4	University Square	8	Basmanj Road	12

The names of the regions and their corresponding numbers considered in this study are recorded in Table 1. In this study, the number of parks to be located in the city is set to 3. For this purpose, after conducting necessary investigations, 12 areas of the city have been identified as candidate points, which are illustrated in Figure 1. The distances between them have been selected using Google Maps for analysis. Finally, from these 12 areas, parks will be established in 3 areas selected based on the problem-solving results.

Figure 1

Map of Tabriz City Along with Candidate Points for Location

Table 2

Distance Matrix

i \ j	1	2	3	4	5	6	7	8	9	10	11	12
1	-	5.1	9.3	15.1	5.7	6.9	4.7	10.2	9	13.4	14.5	16.1
2	-	-	5.7	12.4	3	4.7	4.6	7.9	6.9	12.3	10.3	13.6
3	-	-	-	4.4	8.8	7.9	11.1	5.9	8.7	9.2	7.6	10.7
4	-	-	-	-	9	6	11.8	5.1	7.8	6.7	4.7	5.8
5	-	-	-	-	-	2.3	2.2	6.1	5.6	8.2	9.5	11.6
6	-	-	-	-	-	-	4.4	4.4	2.7	6.4	8	7.6
7	-	-	-	-	-	-	-	7.8	4.8	9.9	11.2	11.8
8	-	-	-	-	-	-	-	-	5	4.9	5.6	6.3
9	-	-	-	-	-	-	-	-	-	6.6	7.9	7.4
10	-	-	-	-	-	-	-	-	-	-	4	6.3
11	-	-	-	-	-	-	-	-	-	-	-	3.5
12	-	-	-	-	-	-	-	-	-	-	-	-

Table 3 shows all the demand region elements as fuzzy random variable for the use of family parks.

Table 3

Demand for points for the use of family park

Number of Regions	$w_i^{(0)}$	$w_i^{(1)}$	$w_i^{(2)}$	β_i	γ_i
1	400	450	9	30	60
2	150	200	3	10	10
3	400	450	4	15	20
4	300	350	4	40	30
5	500	550	8	50	30
6	450	500	10	40	30
7	200	250	4	20	10
8	600	650	8	50	40
9	250	300	6	15	30
10	350	400	6	30	50
11	500	550	5	50	60
12	100	150	7	20	20

3.1. Results from the Case Study

To determine the suitable locations of family parks, the models for P-center problem in both fuzzy and fuzzy random states using the GAMS software were employed. The modeling conducted for the problem has coded feasibility and necessity criteria for both states based on the parameters discussed. The obtained values for the objective function were recorded for further analysis and are presented in Tables 4, 5, and 6.

3.1.1. Determining the Optimal Point in the Model Based on Feasibility Criteria

To determine an optimal point in the fuzzy state, where the goal is to achieve the maximum level of feasibility, the optimal point $\eta = 0.9$ is selected at an objective function of 2271.450.

In the fuzzy random state, if the optimal solution is defined as the minimum value and no other strategy is considered, the objective function values are compared in Table 5. From the comparison, the lowest value present in the table, which represents the objective function at the optimal point, is obtained. This value, specified in Table 5, is determined as $\boldsymbol{\eta} = \mathbf{0.1}$ and the probability level $\boldsymbol{\theta} = \mathbf{0.3}$ equals 2126.28, which is at the level of feasibility.

It is noteworthy that in the fuzzy state, only the conditions of ambiguity using the level of possibility influence the determination of the optimal point. However, in the fuzzy random state, both ambiguity and the randomness of events enter the decision-making process, using levels of possibility and probability.

Table 4

The values for Objective Function at the Fuzzy State

η	The model based on the necessity criteria	The model based on possibility criteria
0.1	2138.500	2271.450
0.2	2162.000	2262.900
0.3	2185.500	2254.350
0.4	2209.000	2245.800
0.5	2232.500	2232.500
0.6	2245.800	2209.000
0.7	2254.350	2185.500
0.8	2262.900	2162.000
0.9	2271.450	2138.500

Table 5

The Values for Objective Criteria in the Model Based on the Possibility Criteria-Fuzzy Random State

θ \ η	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	2173.866	2182.416	2155.42	2178.92	2202.42	2216.616	2225.166	2233.716	2242.266
0.2	2183.898	2192.448	2200.998	2209.548	2212.76	2226.648	2235.198	2243.748	2252.298
0.3	2126.28	2149.78	2208.294	2216.844	2225.394	2233.944	2242.494	2251.044	2259.594
0.4	2132.625	2156.125	2179.625	2223	2226.625	2240.1	2248.65	2257.2	2265.75
0.5	2138.5	2162	2185.5	2209	2232.5	2245.8	2254.35	2262.9	2271.45
0.6	2144.375	2167.875	2191.375	2214.875	2238.375	2261.875	2285.375	2308.875	2332.375
0.7	2150.72	2174.22	2197.72	2221.22	2244.72	2257.656	2291.72	2315.22	2338.72
0.8	2158.24	2181.74	2205.24	2228.74	2252.24	2264.952	2273.502	2282.052	2346.24
0.9	2168.58	2192.08	2215.58	2239.08	2262.58	2274.984	2283.534	2292.084	2300.634

Table 6

The Values for Objective Criteria in the Model Based on the Necessity Criteria-Fuzzy Random State

θ \ η	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	2242.266	2233.716	2225.166	2216.616	2202.42	2178.92	2155.42	2182.416	2173.866
0.2	2252.298	2243.748	2235.198	2226.648	2212.76	2209.548	2200.998	2192.448	2183.898
0.3	2259.594	2251.044	2242.494	2233.944	2225.394	2216.844	2208.294	2149.78	2126.28
0.4	2265.75	2257.2	2248.65	2240.1	2226.625	2223	2179.625	2156.125	2132.625
0.5	2271.45	2262.9	2254.35	2245.8	2232.5	2209	2185.5	2162	2138.5
0.6	2332.375	2308.875	2285.375	2261.875	2238.375	2214.875	2191.375	2167.875	2144.375
0.7	2338.72	2315.22	2291.72	2257.656	2244.72	2221.22	2197.72	2174.22	2150.72
0.8	2346.24	2282.052	2273.502	2264.952	2252.24	2228.74	2205.24	2181.74	2158.24
0.9	2300.634	2292.084	2283.534	2274.984	2262.58	2239.08	2215.58	2192.08	2168.58

To determine the change process of objective function, with different values for possibility and probability levels, as well as the intuitive identification of the optimal point, 3D diagrams can be used. Two examples of such diagrams are presented as follows for further consideration.

Figure 2

3D Diagram for Objective Function Values in the Model, Based on the Possibility Criteria-Fuzzy Random State

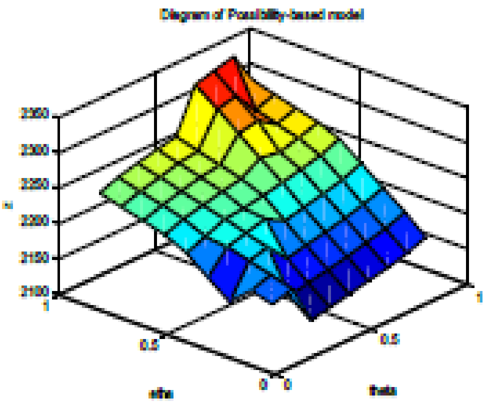
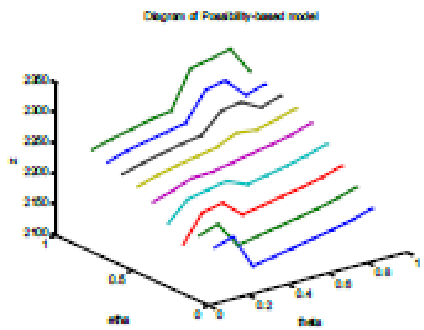


Figure 3

The Change Processes for Objective Function for a Specific η, Based on the Possibility Criteria-Fuzzy Random State.



In Figure 3, the optimal point is indicated by an ellipse. It can also be observed graphically that the optimal point occurs at η=0.1 and θ=0.3. For the case of η=0.1 and θ=0.3, family parks are located at points 4, 7, and, 8. Consequently, the demand from other points is also met according to the allocation made, minimizing the maximum weighted distance in the network.

In determining the optimal point, there can be several scenarios and strategies. In this section, four scenarios are defined, and the results obtained are presented:

- η=0.9, θ=0.1, Objective function value = 2242.26
- η=0.1, θ=0.9, Objective function value = 2168.58
- η=0.5, θ=0.1, Objective function value = 2202.42
- η=0.1, θ=0.5, Objective function value = 2138.50

1. Achieving the maximum level of possibility and the minimum objective function value.
2. Achieving the maximum level of probability and the minimum objective function value.
3. Achieving the average level of possibility and the minimum objective function value.
4. Achieving the average level of probability and the minimum objective function value.

3.1.2. Determining the Optimal Point in the Model Based on the Requirement Criterion

To determine the optimal point in the fuzzy case, if the objective is to achieve the maximum level of possibility, using Table 4, the optimal point is selected at η=0.9, with an objective function value of 2138.500. In the case of random fuzziness, if only the minimum solution is considered optimal, the objective function values in Table 6 are compared, and the optimal value is determined to be 2126.28, which is achieved at the level of possibility η=0.9 and the level of probability θ=0.3. This value is indicated in Table 6. This optimal point is the same as the point obtained in the model based on possibility. The similarity of this point is due to the specific shape of constraint (1) in both models.

Figure 4

The Change Processes for Objective Function for a Specific η, Based on the Necessity Criteria-Fuzzy Random State

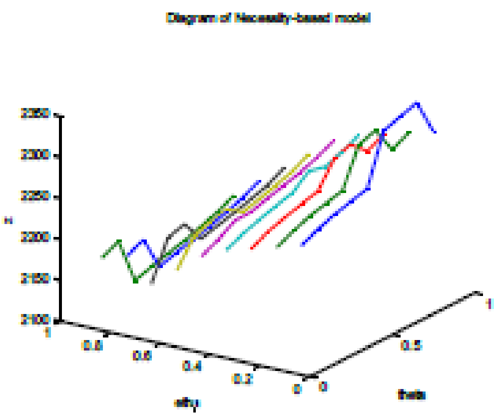
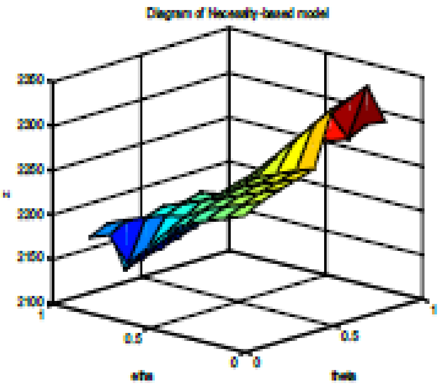


Figure 5

3D Diagram for Objective Function Values in the Model, Based on the Necessity Criteria-Fuzzy Random State



The scenarios presented in section 3-1-1 can also be examined in this section, resulting in the following:

1. η=0.9, θ=0.3, Objective function value = 2126.28
2. η=0.9, θ=0.9, Objective function value = 2168.58
3. η=0.5, θ=0.1, Objective function value = 2202.42
4. η=0.9, θ=0.5, Objective function value = 2138.50

4. Conclusion

In this article, a method of fuzzy random linear programming with fuzzy random variables was proposed for the P-center problem. Initially, the FLP model was formulated and transformed into an LP model using possibility and necessity values. The FLP model was used to derive the FRLP relationship, and finally, the FRLP model was also modeled based on the theory of possibility and necessity for optimistic and pessimistic decision-makers. In this article, four theorems were presented to convert the obtained models into deterministic linear programming models. In the final stage, a case study was conducted in the city of Tabriz to evaluate the usefulness of the proposed model. This study showed that with the fuzzy model, it is possible to approach reality and incorporate the levels of possibility and probability of decision-makers, as well as their optimism and pessimism, into the model and the determination of an optimal point. Since uncertainty is an inseparable part of the problem and each parameter requires the application of this uncertainty, it is suggested that fuzzy modeling be applied to facility location problems, including covering problems, P-median problems, etc., so that the results obtained from solving them are closer to real-world results.

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