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Multi-Period Portfolio Selection: Balancing Return and Squared Value at Risk Objectives

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A R T I C L E I N F O

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A B S T R A C T

In this paper, we have modeled and optimized the multi-period stock portfolio by considering variance heterogeneity and determining the optimal number of stock packages. This model seeks to maximize the return and minimize the risk of the investment portfolio using the squared value at risk. Due to the investment portfolio in this research is based on predicted values; therefore, autoregressive modeling and variance heterogeneity have been used to predict stocks returns. Prediction is done with Python software. The linearized mathematical model for optimizing the portfolio in each period was solved using GAMS software. Furthermore, three stock portfolio designs, including predicting returns and optimizing periodic portfolio, a random portfolio, and a combination of low-risk and high-yield cases have been investigated. In two designs, the random portfolio and the portfolio with 5 high-return and 5 low-risk stocks, with the increase in the risk rate level, the annual return increases, which indicates the consistent relation between risk and return. In the periodic portfolio, this trend has been observed up to 20% risk level, while at 25% risk, there has been a decrease in return. The periodic portfolio has shown more fluctuations in profitability, while the combined approach and the random portfolio have had a more stable trend in increasing profitability with increasing risk.

Introduction

Optimal investment portfolio is one of the significant and challenging issues in the field of financial management and investment, aiming to create a combination of financial assets that yield high returns with low risk. Generally, more profitable investments are usually associated with higher risk, while lower-yield investments come with lower risk. Therefore, selecting the

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most desirable investment in terms of profitability while considering an acceptable level of risk is a challenging task. To model an investment portfolio, it is necessary to use appropriate criteria (objective functions) for measuring risk and return. Various criteria such as variance (in the Markowitz model), Value at Risk (VaR), and Conditional Value at Risk (CVaR) have been utilized to measure the risk of stock portfolios. Value at Risk is one of the effective metrics in risk management and optimizing investment portfolios, measuring the maximum potential loss for a portfolio at a specified confidence level. In practice, a specific number of shares for each asset is purchased in stock market investment. Therefore, the main decision variable in portfolio selection models is the number of each asset. However, generally, the weight of the asset is used as a continuous variable instead. If the number of shares purchased from each asset is considered as the decision variable, it leads to a more realistic model that can be directly used as stock purchase orders without the need for additional transformations. Additionally, considering discrete variables makes it easy to incorporate constraints related to the minimum or maximum number of shares that can be purchased into the model.

One of the critical issues in stock portfolio management, especially in multi-period scenarios, is controlling the costs associated with buying and selling stocks. Each purchase or sale incurs transaction costs, which include fees and taxes. Considering rebalancing constraints presents a new perspective on managing the costs related to buying and selling stocks and rebalancing the investment portfolio. These constraints help manage the number of transactions and, consequently, their costs. Moreover, in a multi-period scenario, the cost of forming a new portfolio also depends on the current portfolio's status. Another issue is forming a future portfolio based on price forecasts or based on past returns and risks, which in traditional models is often based on past returns and risks. This can lead to significant errors and even bankruptcy in cases where the holding period of the portfolio is short. In traditional price forecasting models, a limiting assumption is the constancy of the variance of error terms, which is usually not valid in real conditions, at least over specific time intervals. Therefore, the modeling of non-constant variance of error terms (heteroscedasticity) should also be considered. Heteroscedasticity refers to the variability of asset return variances over time, which is one of the characteristics of financial markets.

The aim of this paper is to model a multi-period stock portfolio where the return on the portfolio's assets is maximized over several periods, and the risk is minimized using the squared VaR criterion. To this end, the problem is defined as a two-objective optimization model for each period and solved using linearization of the model and applying an exact method in GAMS software, considering the current portfolio status as input to the model. This model also incorporates rebalancing constraints. The decision variable in this model is the number of bundles obtained from each asset. In this paper, GARCH models are used to model the heteroscedasticity of error variance and autoregressive models for forecasting the future prices of assets. Subsequently, Section 2 provides a brief review of the literature and research background. Section 3 describes the research methodology. Sections 4 and 5 present the research findings and conclusions, respectively.

Literature Review and Research Background

The Markowitz portfolio optimization model is one of the foundational and most popular portfolio optimization models. Most portfolio optimization models, even in recent years, are based on the Markowitz model, with differences arising in risk measurement methods, types of constraints, approaches to multi-objective handling, and solution methods. Acknowledging this, this paper reviews only a selection of recent articles in this section, avoiding repetition of the foundational models reported in most papers. Ferreira and Cardoso (2021), in a paper titled "Mean-CVaR Portfolio Optimization Approaches with Cardinality Constraints and Rebalancing

Process," evaluated and compared nonlinear multi-objective portfolio optimization models and linear single-objective models with integer and continuous decision variables. This paper considers two different classes of nonlinear bi-objective and single-objective linear models (which are approximations of the first model) with the goal of maximizing expected return and minimizing CVaR, subject to rebalancing and variable cardinality constraints. To evaluate the performance of the models, historical daily price series of 53 assets from the Brazilian stock index (Bovespa) from January 2013 to December 2016 were used. They employed exact branch-and-bound methods and a multi-objective genetic algorithm (NSGA-II) to solve the problem. According to the findings of this paper, in-sample analysis indicated that the exact models provided a set of solutions with greater coverage and a Pareto frontier closer to the optimal frontier. At the same time, the numerical superiority of the exact methods was minimal. Therefore, in many cases, the advantage of using metaheuristic methods may be greater due to shorter execution times. Out-of-sample analysis indicated the stability of portfolio optimization models and similar behavior of financial returns for different transaction cost levels, suggesting that portfolio optimization using historical price series with daily granularity and monthly rebalancing entails lower risk compared to using hourly granularity and daily rebalancing. However, using granularity higher than hourly may yield higher returns in short time periods. Nevertheless, employing rebalancing is justifiable given the robustness of the method against various transaction costs (Ferreira & Cardoso, 2021). Raei et al. (2020) proposed a model for more accurately measuring risk in stock portfolios. They sought to address the question of whether calculating CVaR using modeled variance can aid in achieving an optimal portfolio derived from the Mean-CVaR method. They formed optimal stock portfolios using the Mean-CVaR method based on adjusted daily closing prices of 30 listed companies from the beginning of 2005 to August 2016 and employed GARCH, T-GARCH, and E-GARCH models to model the variance changes in stock returns. A comparison of the performance of variance calculation methods based on the Sharpe ratio and results obtained through statistical analyses, paired comparisons, and Wilcoxon tests shows that at a 95% confidence level, portfolios constructed using three methods of modeled variance significantly outperformed those obtained from historical (constant) variance (Raei et al., 2020).

In 2019, Nguyen et al. (2019) measured the non-linear risk of investment portfolios using the CVaR metric. They utilized daily closing price data of 30 major U.S. companies to address the optimization problem with non-linear optimization methods, employing genetic algorithms for its resolution. Based on the analyses conducted in their paper, optimized portfolios with Mean-CVaR objective functions can yield higher returns and lower risk in conditions where the market exhibits non-linear risk. Furthermore, the objective functions performed better for portfolios that included high-risk stocks with sudden shocks. Nguyen and Huynh (2019) examined the performance of a combined model of Copula, GJR-GARCH, EVT, and CVaR for optimizing stock portfolios using daily data from stock indices of six ASEAN member countries, considering the dependency structure between them from January 2001 to December 2017. They employed a local search method for portfolio optimization. According to the results, the proposed model outperformed traditional portfolio optimization models, as traditional models defined by variance risk did not fully account for non-linear risk, leading to the formation of portfolios with greater non-linear risk. Additionally, considering the dependencies between assets reduced the portfolio's sensitivity to market shocks. This is attributed to the protection of assets against market shocks due to the positive dependency among stocks (Nguyen & Huynh, 2019).

A new method for predicting stock prices using machine learning and portfolio optimization (considering the mean-variance approach) was presented by Chen et al. (2021). Their research data included daily stock prices of 50 companies in the Chinese financial market from 2010 to 2019. The preprocessing of data involved converting data to daily change ratios, removing outliers and incomplete data, and extracting robust features from the data using the Huber method. They utilized recurrent neural networks and convolutional neural networks for stock price prediction, along with genetic algorithms for portfolio optimization based on mean and variance return metrics.

Yu and Liu (2021) investigated an important issue in the realm of personalized investment portfolios. This issue concerns how to construct an optimal investment portfolio that offers high returns and low risk, taking into account the risk tolerance of investors. To address this issue, they proposed an optimization model using daily data from 100 stocks in the Chinese financial market from 2015 to 2019, using CVaR as the risk measurement criterion. Initially, risk tolerance was determined using a fuzzy composite evaluation method based on the demographic characteristics of investors, utilizing a questionnaire to gather information regarding gender, age, education, income, investment experience, and investment goals. Subsequently, using a fuzzy model, each investor was categorized into one of three risk tolerance categories: high, medium, and low. Following this, the time series of returns was estimated using the GARCH model, and the joint distributions of returns between assets were described by the Copula model based on historical data. Future return scenarios were generated through Monte Carlo simulations based on the results of the Copula-GARCH combined model to estimate CVaR. The Mean-CVaR portfolio optimization model for creating personalized investment portfolios was a mixed-integer linear model, solved using the PSO algorithm.

Modeling the Problem and Solution Method

In this research, a multi-period stock portfolio selection problem is modeled. At the beginning of the first period, there is a portfolio with an equal number of shares. The stock prices for the first period are predicted, and then the mathematical model for portfolio selection is executed. Based on the model results, the portfolio is formed and maintained until the end of the holding period. At the end of the first period, this process is repeated to form and maintain the portfolio for the next period. Therefore, the two main modules of this problem include price forecasting and solving the mathematical model for stock selection in each period. The details of these modules and the research methodology are presented below.

Stock Price Forecasting Considering Heteroscedasticity

In financial markets, the phenomenon of heteroscedasticity is an undeniable reality. This phenomenon refers to conditions where the variance of error terms is not constant over time. In this research, the GARCH method is used to model heteroscedasticity. Additionally, for modeling and forecasting stock returns, the autoregressive model of order p AR(p) is utilized. In this model, it is assumed that future values of a variable can be predicted using its past values. To leverage the advantages of both methods, a combination of AR and GARCH models is employed. This combined approach allows for simultaneous modeling of both the mean trend (with the AR model) and the variance changes (with the GARCH model). All these processes are implemented and executed using the Python programming language. To validate the written Python code, data (simulation) is generated, using existing relationships and determining model coefficients. Then, the produced data is input into Python to compare the output coefficients with those defined for data generation. The analysis indicates no significant difference between the original coefficients and those estimated by the Python code, indicating the correctness of the implemented model in Python.

Mathematical Model of the Investment Portfolio Selection Problem

The mathematical model for portfolio selection has two objective functions: maximizing the return of the investment portfolio and minimizing the squared Value at Risk (VaR), including budget constraints and rebalancing the investment portfolio while considering transaction costs. This model is a nonlinear model that has been linearized by the same authors. The details of this model and its validation method are discussed in the paper titled "Linearization of the Portfolio Selection Model with Return and Squared Value at Risk Objectives."

Scenario Design Methodology

a. Scenario of Return Forecasting and Periodic Portfolio Formation: In this research, for 6 stocks over a 60-day working period, considering 5-day intervals, the predicted returns and the stock portfolio are formed. For this purpose, daily data for each stock from the beginning of 2023 is considered, and the logarithmic returns are calculated. The return data for each 5-day period is input into Python. Among the outputs from Python, which include various models for predicting 5-day returns while considering heteroscedasticity, a model is selected that has the lowest AIC and BIC parameters. In other words, these two criteria are used to determine the optimal order of the AR(p) and GARCH(p, q) models. Then, to form the portfolio in each 5-day interval, the mean and variance for the 6 stocks are calculated using the predicted 5-day data and used as inputs for the mathematical model. The exact solution of the mathematical model is performed using GAMS software for 12 periods. Finally, the profit obtained from this 60-day forecast is calculated and serves as the basis for model comparison.

b. Scenario of Random Stock Portfolio Formation: Among 20 selected stocks in the market, 6 stocks are chosen completely at random. For each of these 6 selected stocks, the logarithmic returns are calculated using daily closing price data in 2023. Subsequently, the input parameters for the GAMS model, including the mean return, variance, and variance-covariance matrix for each stock, are computed. This methodology allows for the creation of a diverse and random sample of stocks that can adequately represent the overall market.

c. Scenario of Portfolio Formation Using 5 Low-Risk and 5 High-Yield Stocks: In this approach, to form a diversified and balanced investment portfolio, 10 stocks are selected from the 20 stocks available in the stock exchange. This selection includes 5 low-risk stocks and 5 high-yield stocks. The criterion for selecting the 5 low-risk stocks is the minimum variance of returns. The criterion for selecting high-yield stocks is the maximum average return. This approach provides a balanced combination of low-risk and high-yield stocks in the selection and formation of the investment portfolio.

Numerical Results of the Model

The stock price data was collected from the Tehran Stock Exchange website, and the daily closing prices of 20 companies active in various industries of the Tehran Stock Exchange were examined over the period from the beginning to the end of 2023. The stocks Ranfor, Madaran, Azar, Akhbar, Sadasht, Dasouh, Katabess, Shavan, Komaseh, and Vanaft were identified as high-volatility stocks, while Famili, Foolad, Fabahonar, Fakhrooz, Shepna, Shebandar, Kachad, Kazar, Zagros, and Seshargh were selected as low-volatility stocks.

Results of the Return Forecasting and Periodic Portfolio Formation Scenario

Based on the steps outlined regarding this model, after obtaining the predicted returns for each stock, the inputs for the GAMS model for executing each of the 12 periods were calculated. To investigate the effect of variable risk levels on the profitability of the model and its sensitivity analysis, periodic portfolios were formed at 5 risk levels: 5%, 10%, 15%, 20%, and 25%, with a total investment budget of 90 million rials. It is worth noting that the budget for each period was calculated considering the reinvestment of the previous period's profit. A sample of the model execution results for 12 periods at a 5% risk level is presented in Table 1.

Table 1. Results of Model Execution for 12 Periods at a 5% Risk Level

Annual Profit Percentage	Profit from 12 Periods (60 Working Days)	Final Profit from 12 Periods (Rials)
44.28%	10.54%	9,488,189

The results obtained from analyzing the contents of each portfolio and the number of asset bundles (each bundle equivalent to 5 million rials) available from each stock for risk levels of 5%, 10%, 15%, 20%, and 25% are examined, with a sample for the 5% risk level presented in Table 2. The profitability results of forming periodic portfolios for these 5 different risk levels are reported in Table 3 and Figure 1.

Table 2. Number of Bundles Available from Each Stock in the Periodic Portfolio at 5% Risk

t=12	t=11	t=10	t=9	t=8	t=7	t=6	t=5	t=4	t=3	t=2	t=1	Symbol
0	0	0	0	0	0	0	0	0	0	0	0	Ranfor
0	5	2	15	5	5	10	15	14	14	0	11	Madaran
3	0	3	2	0	4	5	0	0	0	0	2	Vaazar
14	14	11	0	2	4	3	3	3	3	0	4	Sadasht
0	0	0	0	0	0	0	0	0	0	0	0	Dasouh
0	0	0	0	0	0	0	0	0	0	0	0	Fakhrooz

Table 3. Results of Profitability from Forming Periodic Portfolios for 5 Different Risk Levels

25%	20%	15%	10%	5%	**Risk Level**
51.28%	72.23%	48.48%	45.68%	44.28%	**Annual Profit Percentage**
10,988,784	15,477,616	10,388,546	9,788,308	9,488,189	**Profit Amount (Rials)**
-4.99%	5.65%	0.67%	0.33%	-	**Profit Growth Percentage with Increased Risk**

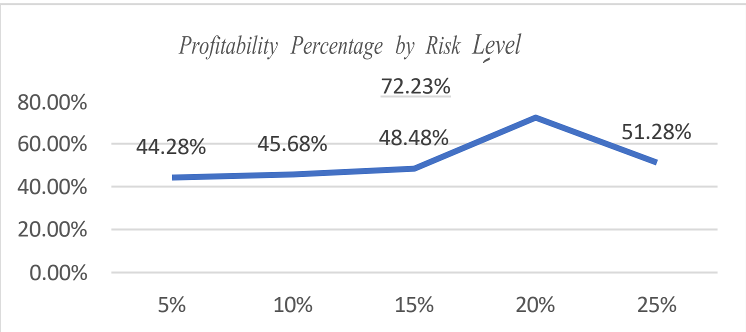

Figure 1. Profitability Percentage by Risk Level for Periodic Portfolio

Results of the Random Stock Portfolio Formation Scenario

The stocks Sadasht, Ketabas, Vanaft, Fabahonar, Shebandar, and Kachad were randomly selected in this approach, and their optimal investment portfolio was formed considering a budget of 90 million rials across 5 different risk levels over a 60-day period using GAMS software. The results are presented in Table 4.

Table 4. Results of Random Stock Portfolio at 5 Different Risk Levels

Risk Level (%)	Annual Profit Percentage	Daily Profit Percentage	Daily Profit Amount (Rials)	Kachad	Shebandar	Fabahoner	Vanaft	Ketabas	Sadasht
5%	-	24.595%	845.83	0	0	0	1	1	3
10%	12.053%	36.648%	934.124	0	0	0	1	1	5
15%	8.063%	44.710%	421.152	0	1	0	0	1	6
20%	9.455%	54.165%	653.184	0	0	0	2	1	7
25%	4.556%	58.721%	186.200	0	0	0	0	1	8

Results of Portfolio Formation Using 5 Low-Risk and 5 High-Yield Stocks

Based on the average returns and variances of the 20 selected stocks, the 5 stocks Madaran, Akhbar, Sadasht, Komaseh, and Seshargh exhibited the highest average returns, while the 5 stocks Ranfor, Madaran, Dasouh, Fakhrooz, and Zagros had the lowest variance. As a result, with Madaran being common in both groups, a total of 9 stocks were used to form the portfolio. Based on a budget of 90 million rials and this approach, the model was solved at 5 risk levels, and the results are presented in Table 5.

Table 5. Results of Portfolio Formation Using Low-Risk and High-Yield Stocks at 5 Different Risk Levels

Annual Profit Percentage compared to risk growth	Annual Profit Percentage	Daily Profit Amount (Rials)	SeShargh	Zagros	Fakhrooz	Komaseh	Dasouh	Sadasht	Akhbar	Madaran	Ranfor	Risk Level
-	32.762%	688.111	1	0	0	0	1	3	0	2	1	5%
13.064%	45.826%	225.156	2	0	0	0	0	4	0	2	3	10%
10.702%	56.528%	708.192	2	0	0	0	1	5	0	3	3	15%
8.493%	65.021%	661.221	3	0	1	0	0	6	0	4	0	20%
5.566%	70.587%	635.240	3	0	0	0	0	6	0	4	4	25%

Conclusion and Summary

In this article, three methods for forming stock portfolios and forecasting returns in the Tehran Stock Exchange are examined. These methods include periodic return forecasting and portfolio formation, random stock portfolio formation, and portfolio formation using low-risk and high-yield stocks. To optimize all three methods, a linearized mathematical model and GAMS software were used. In both the random portfolio approach and the approach using 5 high-yield stocks and 5 low-risk stocks, annual returns increase with higher risk levels. This indicates a positive relationship between risk and return, which aligns with financial theories. In the periodic portfolio, this trend continues up to a risk level of 20%. However, at a risk level of 25%, a significant decrease in returns is observed due to the simultaneous consideration of the budget and the value of the asset bundle, which imposes a high constraint on the model. Despite the model's efforts to achieve higher profits, it fails to realize greater profitability and to purchase riskier assets. Additionally, in terms of annual profit percentage, the portfolio with 5 high-yield and 5 low-risk stocks has shown greater profitability at all risk levels. The periodic portfolio exhibits more volatility in profitability, while the combined and random portfolio approaches demonstrate a more stable trend in increasing profitability with rising risk.

One limitation of this research is the consideration of capital and budget solely for investment in stocks. In situations where investment in stocks is not possible, or in cases where the returns of all stocks in the portfolio are negative (resulting in no portfolio being formed, as seen in the second period of the periodic portfolio model), the capital remains in cash and is not invested elsewhere. This leads to idle capital, which can negatively impact the overall return on investment. Future research could consider investments in other assets as well. Additionally, this study has overlooked dividend income from stocks. Given that the forecasting periods in this research are short-term and 5 days long, the effect of dividends can be disregarded. However, this could influence the accuracy of return calculations, especially for stocks with high dividend yields. Therefore, another suggestion for future research is to include this aspect in dividend calculations. Furthermore, considering the fixed 5-day periods for forecasting and portfolio formation, this model could be developed by varying the portfolio formation periods and optimizing the investment horizon.

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