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Robust Mathematical Modeling for Air Multiple Logit Design in Flight Route Selection

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ABSTRACT

This study focuses on designing an air multiple logit model using robust mathematical modeling for flight route selection. The proposed model effectively contributes to reducing costs and improving the efficiency of flight routes by incorporating various factors, including aircraft departure costs, flight crew salaries, flight duration between two cities, and other parameters such as passenger capacity per route. The designed model was formulated and solved using GAMS software. The global optimal solutions for the three objective functions of the proposed model were calculated. Based on the obtained results, the first objective function is 182.8240 monetary units, the second is 540.6540 units, and the third is 200.369 monetary units. This model offers the necessary flexibility for optimal flight route management by accounting for uncertainties and market dynamics. The results indicate that implementing this robust model leads to an increase in aircraft departure costs with an increase in samples, while the total objective function value remains stable. Furthermore, flight crew salaries remained constant with increasing samples, and this did not result in an increase in the total cost function value. Therefore, this research underscores the importance of optimal decision-making in minimizing no-shows and superfluous costs.

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Introduction

In the aviation industry, flight delays represent one of the most critical challenges. Furthermore, every airline is obligated to operate specific flights, which inherently necessitates a series of pre-flight services (Sun et al., 2024). Airport services encompass a set of interconnected activities that collectively ensure the necessary conditions for a safe flight. These activities include, but are not limited to: passenger services, cargo handling, loading, and ramp control. Globally, airport ground handling services are currently provided either directly by the airlines themselves or by specialized service companies (often referred to as "ground handlers" or "service support"). Pre-flight services, commonly known as ramp services, refer to the comprehensive array of operations performed on an aircraft during its turnaround time the interval between its arrival at the gate and its departure (Bergantino et al., 2020). Consequently, for each outgoing flight, a defined set of services must be provisioned, and these services can significantly contribute to flight delays (Wandelt et al., 2025).

In light of these challenges, managers and officials are actively seeking solutions to minimize delays. Furthermore, owing to the complexities of flight service scheduling and the presence of numerous constraints and influencing parameters, achieving an optimal demand plan is feasible through mathematical modeling (Abolghasemian et al., 2024). The four-stage process is widely recognized as one of the most common methods for demand analysis in transportation planning. Trip generation models, typically representing the third stage of this process, determine the modal split for urban journeys and prepare the necessary information for trip assignment, constituting the final stage of the four-stage process. The trip generation stage plays a pivotal role in public transportation policymaking due to its inclusion of policy-sensitive variables (Kim, 2020). Relative to other trip generation models, discrete choice models have garnered significant attention and advanced considerably in recent decades (Khan et al., 2024). The structure of these models is probabilistic, mathematically modeling the decision-maker's behavior and their endeavor to maximize the utility derived from a given choice. Stochastic utility models have been employed in transportation research for over 50 years (Sun & Wang, 2023). Initially, almost all applications relied on the multinomial logit model. The multinomial logit model is considered the simplest, most fundamental, and most widely used discrete choice model. Its popularity stems from providing a closed-form expression for the probability of choosing alternatives, which is straightforward to interpret (Akbaba, 2023). The structure of the logit model was first utilized by Berkson (1944). This model gained significant attention from researchers in the 1970s following McFadden’s (1974) seminal work. The introduction of the Generalized Extreme Value (GEV) family of models alleviated some of the inherent limitations of the standard logit model, with nested logit models being among the most prominent examples (Pamucar et al. 2020). Two other types of discrete choice models, often recognized for their flexibility, are the multinomial probit model and the mixed multinomial logit model (Khaleghi & Eydi, 2023). Probit models possess a highly complex structure, rendering their application considerably costly. Furthermore, probit models lack closed-form expressions (e.g., exponential, fractional), further complicating their practical implementation. The mixed logit model is capable of providing numerous behavioral and physical interpretations. Similar to the probit model, the

mixed logit model has been recognized for many years; however, due to the absence of a closed-form expression for computing choice probabilities and the necessity of using simulation methods for its estimation, researchers widely adopted it at first during the last two decades, following advancements in high-speed computing technology (Jiang et al., 2023). The initial applications of the mixed logit model were in automobile demand models, notably employed by Cardell and Dunbar (1980) and McFadden (1974), demonstrating that in the mixed logit model, the behavior of various random utility models could be approximated by specifying appropriate distributions for the coefficients' characteristics within the utility function, with the type of these distributions being determined to better align with reality. The choice of distribution type for randomly distributed coefficients plays a crucial role in the properties of a mixed logit model (Wong et al., 2023).

According to above statement, there are some challenges associated with robust mathematical modeling for air multiple logit design in flight route selection. Ensuring high-quality, comprehensive data on passenger preferences, historical flight data, and operational constraints can be difficult. Moreover, incomplete or inaccurate data can lead to suboptimal model outcomes. Additionally, the complexity of the multiple logit model increases with the number of variables and interactions considered. Balancing model complexity with interpretability is a significant challenge. The aviation market is subject to rapid changes due to factors such as economic fluctuations, regulatory changes, and shifts in consumer behavior. Hence, developing a model that adapts to these dynamics is complex. Flight route selection involves various uncertainties, such as weather conditions, air traffic, and operational disruptions. The robust modeling of these uncertainties is a significant challenge. Stochastic spatiotemporal models have also been developed to capture complex dynamics in heterogeneous environments (Gorgi et al., 2025). The underlying assumptions of passenger behavior in the logit model may not always hold true in real-world scenarios. Capturing the nuances of human decision-making can be difficult. As the number of routes and variables increases, the computational burden of solving the logit model can become significant. Ensuring that the model runs efficiently while maintaining accuracy is a challenge. Integrating the logit model with other operational systems (e.g., scheduling, pricing, and revenue management) can be complex and requires careful coordination.

Given the aforementioned points, the aviation industry, driven by technological advancements and continuous growth, necessitates the optimal utilization of its resources and flight networks. A crucial challenge in air route management involves the optimal selection of flight routes, which serves to reduce costs and passenger travel time. Simultaneously, multiple criteria must be considered for this selection, including the provision of quality services, pollution reduction, fuel consumption optimization, the minimization of ground, and enhancement of cultural well-being. The primary objective of this research is to develop a multi-objective mathematical model utilizing robust multinomial logit, which is capable of selecting optimal airline routes by accounting for uncertainty and potential variations in diverse influencing factors. This mathematical modeling aims to consider any changes in conditions and constraints for future decision-making, thereby offering robust optimization against uncertainty. Consequently, leveraging the robustness of multinomial logit in the

problem of optimal air route selection, this research can contribute to providing mathematically-based optimization methods and algorithms, ultimately leading to a more effectively organized aviation industry.

Despite the significance of flight route selection, existing multinomial logit models often overlook the stochastic nature of aviation operational variables. There is a lack of robust mathematical frameworks that can integrate these uncertainties into decision-making processes for air route design. Transitioning from deterministic to robust modeling is critical, as it minimizes the sensitivity of optimal route selections to unpredictable operational disruptions, thereby improving both reliability and sustainability in the airline network. This study aims to bridge this gap by proposing a robust mathematical model designed to handle uncertainty in flight route selection. Specifically, the contributions of this research are threefold: (1) formulating a robust multinomial logit design, (2) incorporating stochastic variables to reflect real-world operational constraints, and (3) providing a decision-making tool that enhances efficiency and sustainability in aviation route selection.

The remainder of this paper is organized as follows. Section 2 provides a literature review of prior research studies to identify existing gaps. Section 3 details the proposed problem framework, and section 4 presents the results of applying the problem to a case study. Finally, Section 5 offers a general conclusion along with suggestions for future research.

Literature Review

Jiang et al. (2026) employed a PVAR model to analyze market reactions to the introduction of the C919 jetliner, finding that price and demand fluctuations are transitory, with effects gradually fading as flight operations stabilize. Teshome et al. (2026) identified seven core technological domains in electric mobility and applies growth models (Logistic, Gompertz, Richards) to determine their lifecycle stages. Findings indicate that while foundational technologies are reaching saturation, intelligent control systems and electric drive systems remain in high-growth phases. Wandelt et al. (2025) provided a comprehensive review of the existing literature on networks and their application in air transport, highlighting the significant role network structures play within the aviation industry. The application domains covered in their review encompass efficiency analysis, flexibility, delays, disease spread, airline strategic planning, and operational planning. Ultimately, the review aimed to synthesize findings from various research streams and publication venues, identify potential overlaps and synergies, and report on emerging trends, research gaps, and future directions. Sun et al. (2024) presented a comprehensive review of airline competition within a critical transitional phase of aviation, which is currently confronting substantial challenges related to climate change and sustainability-linked development goals. Their study and its findings are intended not only to contribute to academic discourse but also to offer valuable insights for policymakers and industry practitioners. Wong et al. (2023) proposed a novel data analytics approach for identifying new long-haul routes and evaluating connectivity quality from a global network perspective. Their proposed approach integrates a comprehensive method for analyzing the aviation network, assessing potential route quality, making decisions on routes that warrant selection, and forecasting the long-term and short-term traffic volume of newly selected routes. To this end, seven features were developed for route selection, growth

coverage, volume, and connectivity potential. Jiang and Zhang (2023) investigated the dynamics of entry, survival, and exit of large long-haul aircraft in China's expanding air transport market. Their study focused on two types of wide-body aircraft, the Airbus A380 and Boeing 747-8, as case studies. Using logistic regression and time-dependent Cox regression models, they analyzed the determinants of route entry and survival for these aircraft from 2011 to 2021. The data were collected based on the flight plans of the air routes and their corresponding aircraft.

Akbaba (2023) selected the most effective flight path in their study through data envelopment analysis (DEA) of potential flight paths to evaluate aircraft resources within the airline industry; the study comprised two stages, wherein the first phase determined effective flight paths in three steps using the DEA method, and the second phase identified the intended flight path by evaluating factors such as the number of transit passengers and flight hours, also using the DEA method, from among the most effective routes determined in the first phase. Tanriverdi et al. (2022) proposed a fuzzy triangular framework based on the Best-Worst Method (BWM) with extensive flexibility for establishing priority weights of factors considered in airport selection; their proposed fuzzy BWM model transforms into a superior decision support system by incorporating the interrelationships between features. To this end, they considered eighteen criteria based on five airport aspects: location, physical features, performance, costs, and reputation, with findings indicating location and costs as the most important aspects, and airport costs and handling costs as the most critical factors. Şimşek and Aktürk (2022) presented flexible airline schedule design, aircraft routing, and fleet allocation problems, incorporating uncertain non-cruise times and controllable cruise times; they employed a data-driven method to estimate flight delay probabilities, calculating the airport congestion coefficients necessary for the probability distribution of non-cruise time random variables, and formulated the problem as a bi-objective nonlinear mixed-integer mathematical model with stochastic constraints. Babagolzadeh et al. (2022) analyzed a cargo distribution network structure and logistics decisions under a subsidy scenario, developing a mixed-integer linear programming model that includes time window constraints; their results demonstrated that providing subsidies can effectively reduce overall costs for industries involved in air cargo distribution. Lu et al. (2021) investigated choice hesitation in airport selection within a multi-airport region using three approaches: the multinomial logit model, the random forest algorithm, and deep reinforcement learning; their results indicated that a quarter of travelers exhibit evidence of hesitation in airport selection; while the deep reinforcement learning approach offered the highest accuracy in predicting stated choices, and the random forest algorithm yielded good decision-making results, the multinomial logit model appeared less accurate in capturing preferred choices. Unal et al. (2021) developed a novel mathematical formulation for air fleet scheduling problems in hub systems; their proposed model aims to minimize the total cost of aircraft allocation to flights while adhering to flight sequence, seat capacity, passenger demand, and aircraft availability.

Considering the aforementioned points, the increasing growth of the aviation industry and the imperative to optimize flight routes establish air multinomial logit design as a pivotal tool in flight route selection and management. However, several research gaps in this domain

necessitate more thorough investigation and analysis. Many existing models for air multinomial logit design do not fully account for uncertainty factors and stochastic variables. This oversight leads to inaccuracies in predicting and selecting optimal flight routes, highlighting a palpable need for developing robust models capable of effectively addressing uncertainties. These research gaps underscore the necessity for developing comprehensive and robust models for air multinomial logit design in flight route selection. Bridging these gaps can contribute to improving the efficiency and sustainability of flight routes, ultimately strengthening the aviation industry. Table 1 presents literature review categorization.

Table 1. Literature Review Categorization

Author	Year	Modeling Method	Solution Logic	Main Finding
Jiang et al.	2026	PVAR model	Analysis of market reaction dynamics and the temporal effects of aircraft entry	Price and demand effects are short-term and diminishing.
Teshome et al.	2026	LDA topic modeling	Identification of technological domains and determination of technology life cycle stages	Seven technological domains were identified; some are approaching saturation while others remain in high-growth phases.
Wandelt et al.	2025	Literature review	Review, synthesis, and identification of research gaps	Summarized the role of network structure in air transport across various domains.
Sun et al.	2024	Literature review	Analytical review and explanation of competitive trends in the airline industry	Examined the challenges of airline competition during the transition toward sustainability and climate change.
Wong et al.	2023	Data analytics framework	New route selection, connectivity quality assessment, and demand forecasting	Proposed a multi-stage framework for selecting long-haul routes and assessing connectivity.
Jiang & Zhang	2023	Logistic regression	Analysis of market entry, survival, and exit	Investigated factors affecting the entry and survival of wide-body aircraft in the Chinese market.
Akbaba	2023	Data Envelopment Analysis (DEA)	Efficiency evaluation and selection of the best route	Selected an efficient flight path among alternatives using DEA.
Tanriverdı et al.	2022	Fuzzy Triangular BWM	Multi-criteria decision-making and weighting of criteria	Identified and ranked the most important airport selection criteria.
Şimşek & Aktürk	2022	Bi-objective nonlinear mixed-integer stochastic model	Optimization of flight scheduling, routing, and fleet allocation under uncertainty	Developed a model for stochastic flight times and operational constraints.
Babagolzadeh et al.	2022	Mixed-Integer Linear Programming (MILP)	Optimization of cargo distribution network with time-window constraints	Showed that subsidies can reduce the total cost of the air cargo distribution chain.
Lu et al.	2021	Multinomial logit	Passenger choice modeling and behavioral prediction	Found that some travelers hesitate in airport choice; DRL achieved higher accuracy.
Unal et al.	2021	Mathematical formulation for fleet scheduling	Minimization of fleet allocation cost under capacity and demand constraints	Proposed a new formulation for scheduling and fleet allocation in hub systems.

Method

Considering the established rules and policies, decisions regarding the selection of multinomial logit for pilots, aircraft, and crew scheduling must be made not only to cover all planned flights but also to minimize the total cost. These costs may include the operational expenses of flights by aircraft, crew salaries for each flight, and the costs associated with empty flights for both aircraft and crew.

An empty flight is a possible, though undesirable, occurrence in the airline industry that can pertain to either the aircraft or the flight crew. An empty flight for an aircraft occurs when an aircraft flies without passengers to reach a specific city (e.g., City *A*) from which it will then operate subsequent flights to other destinations. This scenario might arise due to a high

volume of travel demand originating from City A to other cities, or because aircraft already present in City A are technically unable to operate the scheduled flights from that location. Generally, an aircraft empty flight primarily occurs when an unscheduled flight is performed to reposition the aircraft to a destination where it can then transport passengers to another point. An empty flight for the crew refers to instances where crew members fly as passengers from City *A* to City *B* to take over the operation of a flight from City *B* to City *C*. The main reason for this situation is often that pilots based in City *B* are unable to operate the *B*-to-*C* flight due to having reached their daily (or weekly) maximum flight hours limit or being involved in pre-scheduled training courses. Operating empty flights incurs specific costs for the airline. According to company policies, no crew member should fly beyond a defined maximum duration within the flight planning horizon. This established limit is considered independent of the pilot's specific type. Furthermore, another policy stipulates that all aircraft, regardless of type, can operate for a maximum specified time without undergoing maintenance and repair operations; this must therefore be factored into aircraft routing decisions. As is evident, the problem under consideration is a combined issue of aircraft routing, subject to maintenance and repair limitations, and flight crew scheduling (i.e., assigning flight crew to aircraft flights). The objective is to minimize the total cost of operating flights by both crew and aircraft, alongside the cost of performing empty flights. In addition, the proposed model addresses the allocation of aircraft to routes. This allocation model is formulated to simultaneously adhere to existing constraints and maximize two criteria: the total passengers/miles transported and the total profit generated by each aircraft type. Since the decision variables can take on continuous values as well as binary (zero or one) values, the proposed model will be a multi-objective linear mixed-integer model.

Assumptions of the Problem Model

The most important assumptions for solving the problem are as follows:

- The number of pilots, fleet types, and the airline's flight schedule are known and fixed throughout the planning horizon.
- Maintenance and repair constraints for the fleet are specified and independent of the fleet type.
- The costs associated with assigning crew and aircraft to flights are specified.
- Flight time between any two cities is known and independent of the aircraft type.
- Each flight can be operated by more than one crew and one aircraft.
- A fixed and specified cost, independent of crew and aircraft type, is incurred by the airline for empty flights involving either an aircraft or crew.
- No constraints are considered for the return of crew or aircraft to their originating city after their final flight.
- The initial flight for each aircraft and crew originates from their respective base station.
- Every pilot is qualified to operate any type of aircraft.
- Each crew member must not exceed a maximum of U hours of flight duty during the planning period.

Mathematical Modeling

In this section, the mathematical model of the problem is presented. First, the notations used in the model are introduced.

- **Indices and Sets**

(j,i)	Index of node (city)
a	Index of aircraft
c	Index of crew
m	Index of stations where aircraft can be maintained and repaired
p	Index of flight order
d	Index of routes
k	Index of days
N	Set of nodes
A	Set of aircraft
C	Set of crews
M	Set of maintenance and repair hubs
P	Set of flight order indices
D	Set of number of routes
K	Set of number of days

- **Parameters**

$cost_{aijp}$	Departure cost of aircraft a from city i to city j for flight order p
$cost'_{cijp}$	Salary of crew c for operating a flight from city i to city j for flight order p
$cost''_{amp}$	Cost required to repair aircraft a to perform maintenance operations at hub m for flight order p
PM_{ijka}	Demand for flight from city i to city j on day k using aircraft a
R_{ijka}	Selling tickets for a flight between cities i and j by aircraft a on day k
C_{ijka}	Total cost for a flight between cities i and j by aircraft a on day k
DH	Cost of an empty flight for an aircraft
DH'	Cost of an empty flight for a crew
U	Maximum flight hours for each crew member
M	A very large number
T_{max}	Maximum flight hours for each aircraft on a route
C_p	Maximum aircraft capacity for passenger transportation on a route in flight order p
M_e	Maximum number of aircraft that can be used for passenger transportation on a route
N_a	Number of aircraft a
t_{aijk}	Flight duration of each aircraft a on node (i,j) on day k
t'_{amp}	Repair time of aircraft a at station m for flight shift p
B_{amp}	Budgets of maintains of aircraft a that, having performed an unscheduled flight, require maintenance and repair at hub m for flight order p.

- **Variables**

The problem variables are of three types: binary, integer, and continuous.

- **Binary Variables**

x_{aijp} A binary variable that takes the value of 1 if aircraft a flies from city i to city j at flight order p, and 0 otherwise

X_{aijk} A binary variable that takes the value of 1 if aircraft a is assigned to flight j from i on day k, and 0 otherwise

y_{cijp} A binary variable that takes the value of 1 if crew c takes over the flight from city i to city j for flight order p, and 0 otherwise

z_{iap} A binary variable that takes the value of 1 if aircraft a undergoes maintenance operations at city i for flight order p, and 0 otherwise

- **Integer Variable**

θ_{amp} Number of aircraft a that, having performed an unscheduled flights, require maintenance and repair at hub m for flight order p.

- **Continuous Variable**

φ_{ajmp} Time elapsed (or arrival time) by aircraft a to city j on flight order p, provided that repairs are made at station m

➢ **Objectives and Constraints**

$$\begin{aligned}\min z_1 = &\sum_{p\in P}\sum_{a\in A}\sum_{(i,j)\in N} x_{aijp}cost_{aijp} \\ &+\sum_{p\in P}\sum_{c\in C}\sum_{(i,j)\in N} y_{cijp}cost'_{cijp} \\ &+\sum_{(i,j)\in N}\sum_{p\in P}\sum_{a\in A}(x_{aijp}-1)DH \\ &+\sum_{(i,j)\in N}\sum_{p\in P}\sum_{c\in C}(y_{cijp}-1)DH' + \sum_{a\in A}\sum_{m\in M}\sum_{p\in P} cost''_{amp}\theta_{amp}\end{aligned} \tag{1}$$

$$\max z_2 = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{a=1}^{A} PM_{ijka}X_{aijk} \tag{2}$$

$$\max z_3 = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{a=1}^{A} (R_{ijka}-C_{ijka}).X_{aijk} \tag{3}$$

$s.t.:$

$$\sum_{i=1}^{I} X_{aijk} = 1 \qquad j = 1,\dots,J, k\in K, a\in A \tag{4}$$

$$\sum_{j=1}^{J} X_{aijk} = N_a \qquad i = 1,\dots,I, k\in K, a\in A \tag{5}$$

$$\sum_{i=1}^{I} t_{aijk} X_{aijk} \leq T_{max} \qquad j = 1, \dots, J, k \in K, a \in A \tag{6}$$

$$\sum_{(i,j)\in N} x_{aijp} \leq \sum_{(j,i)\in N} x_{ajip} \qquad \forall a \in A, p \in P \tag{7}$$

$$\sum_{(i,j)\in N} y_{cijp+1} \leq \sum_{(j,i)\in N} y_{cjip} \qquad \forall c \in C, p \in P \tag{8}$$

$$\sum_{a\in A} \sum_{p\in P} x_{aijp} \geq 1 \qquad \forall (i,j) \in N \tag{9}$$

$$\sum_{c\in C} \sum_{p\in P} y_{cijp} \geq 1 \qquad \forall (i,j) \in N \tag{10}$$

$$\sum_{a\in A} x_{ajip} = \sum_{c\in C} y_{cijp} \qquad \forall (i,j) \in N, p \in P \tag{11}$$

$$z_{iap} \leq \sum_{j\in N} x_{ajip} \qquad i = 1,2, \dots, I, \forall a \in A, p \in P \tag{12}$$

$$\sum_{p\in P} \sum_{(i,j)\in N} t_{ij} y_{cijp} \leq U \qquad \forall c \in C \tag{13}$$

$$t_{aijk} + z_{aip-1} t'_{amp} - M(1 - X_{aijk}) \leq \varphi_{ajmp} \leq t_{aijk} + z_{aip-1} t'_{amp} + M(1 - X_{aijk}) \quad \forall a \in A, (i,j) \in N, i \neq j, p \in P, m \in M, k \in K \tag{14}$$

$$\sum_{(i,j)\in N} x_{aijp} \leq C_p \qquad \forall a \in A, p \in P \tag{15}$$

$$\sum_{a\in A} X_{aijk} \leq M_e \qquad \forall (i,j) \in N, k \in K \tag{16}$$

$$\sum_{a,A} x_{aijp} \geq y_{cijp} \qquad \forall (i,j) \in N, c \in C, p \in P \tag{17}$$

$$\sum_{a\in A} \sum_{m\in M} \sum_{p\in P} \theta_{amp} Cost''_{amp} \leq B_{amp} \tag{18}$$

$$y_{cijp}, z_{iap}, x_{aijk}, X_{aijk} \in \{0,1\}, \quad \theta_{amp} \in Integer, \quad \varphi_{ajp} \geq 0 \tag{19}$$

In the objective function (1), the first part represents the total costs of operating flights by aircraft, while the second part represents the total costs related to crew for operating flights. The third and fourth parts of the objective function pertain to the costs associated with empty flights when a scheduled flight is operated by more than one crew and one aircraft. In such cases, the company incurs a fixed cost of *DH* for each aircraft, and *DH′* for each crew member. The next two parts relate to the costs incurred for operating unscheduled flights by crew and aircraft, where the company also bears a fixed cost equal to *DH* for the aircraft and *DH′* for the crew, respectively. In the last term, we calculate the cost required to repair each type of aircraft to perform maintenance operations during each flight period. As mentioned above, this objective function aims to minimize all these costs. Equations (2) and (3) represent the second and third objective functions of the problem, respectively. They aim to maximize the total passengers/miles transported and the total profit generated by each aircraft type.

Constraint (4) ensures that each aircraft is assigned to a single route. Constraint (5) guarantees the available inventory count for each aircraft type. Constraint (6) imposes the maximum flight hours for each aircraft type. Constraint (7) ensure that each aircraft of type a assigned to node (i,j) in flight period p is also assigned to node (j,i) for flight order p. Similarly, constraint (8) ensures that for a crew member to be able to fly from a city at flight order p, they must have arrived at that city from another city at flight order p. Constraints (9) and (10) ensure that all scheduled flights must be covered by aircrafts and crews. It is clear that if the left side of constraints (4) and (5) exceeds one, an empty flight has occurred for crew and aircraft. Constraint (11) ensures that an aircraft can perform a flight at order p only if a crew has also taken charge of that flight at the same flight order. According to constraint (12), if an aircraft undergoes maintenance operations at one of the maintenance and repair hubs at a certain flight order, it must have flown from one of the cities to that specific hub at the same flight order. Constraint (13) ensures that no crew member should fly more than U hours within the planning horizon. Constraint (14) calculates the elapsed time for aircraft a to traverse arc (i,j) and arrive at city j on flight order p, assuming maintenance is performed at station m. Here, t'_{amp} is the fixed and predefined maintenance duration for aircraft a at station m. If the aircraft is assigned to arc (i,j) (X_{aijk}= 1), the term $(1-X_{aijk})M$ vanishes, and the constraint reduces to the following equality: $\varphi_{ajmp} = t_{aijk} + z_{aip-1}t'_{amp}$. This equality implies that the arrival time at city j equals the flight duration plus the maintenance time if maintenance was performed at the previous station (z_{aip-1}= 1); otherwise, it equals the flight duration alone. If the aircraft is not assigned to this arc (X_{aijk}= 0), the term $(1-X_{aijk})$ becomes equal to the large number M, sufficiently relaxing both the lower and upper bounds. Consequently, the constraint becomes inactive and imposes no restriction on φ. Constraint (15) ensures that the capacity of each aircraft does not exceed its permissible limit. Constraint (16) ensures that a specified number of aircraft is present on each flight route. Constraint (17) guarantees that if an aircraft is assigned to any flight route, it must perform at least one trip. Constraint (18) guarantees that the total costs resulting from all decisions must not exceed the available budget or cost limit. Constraint (19) specifies the types of decision variables.

Multiple Logit Model

In this study, the route choice behavior of passengers is first represented through a multinomial logit (MNL) framework, and the resulting choice probabilities are then incorporated into the robust optimization model. Let U_{ij} denote the utility of choosing route j for traveler i, which is defined as $U_{ij} = v_{ij} + \varepsilon_{ij}$, where v_{ij} is the systematic component of utility and ε_{ij} is the random error term. The deterministic utility is specified as $v_{ij} = \beta_0 + \beta_1 T_{ij} + \beta_2 C_{ij} + \beta_3 S_{ij}$, where T_{ij}, C_{ij}, and S_{ij} represent travel time, travel cost, and service quality (or any other route attribute used in the study), respectively, and $\beta_0, \beta_1, \beta_2, \beta_3$ are the estimated parameters. Under the MNL assumption, the probability that route j is chosen by traveler i is given by $p_{ij} = \frac{\exp(v_{ij})}{\sum_{k\in J}\exp(v_{ik})}$, where j denotes the set of all available routes. The expected demand for route j is then computed as $D_j = q_i P_{ij}$, where q_i is the total demand

associated with traveler group i. These expected demands are used as inputs to the MILP model selecting the optimal robust flight route configuration under uncertainty. The MILP formulation is presented in the previous section. The objective function and constraints determine the final route selection decisions based on the logit-derived demand estimates.

Robust Mathematical Modeling

As mentioned, the presented model minimizes the total cost of operating flights. Given that the demand is fluctuating or, in other words, accompanied by uncertainty, the values of the objective functions and the problem's optimal variables can significantly differ from those obtained from the current model. Therefore, to reduce this difference, this section aims to make the model robust against uncertain parameters. In a robust optimization model, two types of variables exist: design variables and control variables. Design variables are determined before the realization of probabilistic parameters and cannot be adjusted afterward. Control variables, however, become subject to adjustment once a specific realization of the probabilistic parameters occurs. Furthermore, the constraints of the robust model comprise structural constraints and control constraints. Structural constraints do not include uncertain parameters or variables, whereas control constraints do contain uncertain parameters or variables, based on Mulvey et al. (1995). First, consider the linear optimization model from equations (20) to (23):

$$minz = C^T X + d^T Y_S \tag{20}$$

subject to:

$$AX = b \tag{21}$$

$$B_s X + C_s Y_s = e_s \qquad \forall\, s \tag{22}$$

$$X, Y_s \geq 0 \tag{23}$$

Where X is the vector of design variables and Y is the vector of control variables. S denotes the set of available scenarios, and the probability of occurrence for each scenario is P_s so that $\sum_{s=1}^{S} P_s = 1$. Constraints (21) and (22) represent the structural and control constraints of the problem, respectively.

δ_s are deviation variables and measure the amount of infeasibility of the control constraints under scenario S. The robust mathematical model, including equations (24) to (27), is expressed as follows:

$$\min \sigma(\mathrm{x}, y_1, y_2, \dots, y_s) \ + \omega\, p(\delta_1, \delta_2, \dots, \delta_s) \tag{24}$$

subject to:

$$AX = b \tag{25}$$

$$B_s X + C_s Y_s + \ \delta \mathrm{s} = e_s \qquad \forall\, s \tag{26}$$

$$X, Y_s \geq 0 \qquad \forall\, s \tag{27}$$

Where the expression $\sigma(\mathrm{x}, y_1, y_2, \dots, y_s)$ measures the robustness of the solution and the expression $p(\delta_1, \delta_2, \dots, \delta_s)$ measures the robustness of the model, which can be defined by various functions. For example, $p(\delta_1, \delta_2, \dots, \delta_s)$ is usually considered equal to $\sum_{s \in S} P_s \ \delta_s$. Let the expression $\sigma(\mathrm{x}, y_1, y_2, \dots, y_s)$ to be equal to the sum of the expected value and λ times the variance of the objective function, according to equation (28).

$$\sigma(x, y_1, y_2, \dots, y_s) = \sum_{s\in S} P_s \; Z_s + \lambda \sum_{s\in S} P_s \, (Z_s - \sum_{s'\in S} P_{s'} \, Z_{s'} \,)^2 \qquad (28)$$

Mulvey et al. (1995) suggested by substituting $\sigma(x, y_1, y_2, \dots, y_s)$ into the objective function, a second-degree nonlinear programming model is obtained. The expression $\sum_{s\in S} P_s \, (Z_s - \sum_{s'\in S} P_{s'} \, Z_{s'} \,)^2$, which can lead to an increase in the problem's solution time, can be replaced with the absolute value expression. Therefore, the robustness of the solution changed, as shown in equation (29).

$$\sigma(x, y_1, y_2, \dots, y_s) = \sum_{s\in S} P_s \; Z_s + \lambda \sum_{s\in S} P_s \; |Z_s - \min \sum_{s'\in S} P_{s'} \; Z_{s'}| \qquad (29)$$

The objective function is a nonlinear equation whose minimization $\sigma(x, y_1, y_2, \dots, y_s)$ is equivalent to the linear programming model including equations (30) to (32), expressed as follows:

$$\min \sum_{s\in S} P_s \; Z_s + \lambda \sum_{s\in S} P_s \; (Z_s - \min \sum_{s'\in S} P_{s'} \; Z_{s'} + 2\theta_s) \qquad (30)$$

subject to:

$$Z_s - \min \sum_{s'\in S} P_{s'} \; Z_{s'} + \theta_s \geq 0 \qquad \forall\, s \qquad (31)$$

$$\theta_s \geq 0 \qquad \forall\, s \qquad (32)$$

As a result, the optimization model will be transformed into the following robust model (33).

$$\min \sum_{s\in S} P_s \; Z_s + \lambda \sum_{s\in S} P_s \; (Z_s - \sum_{s'\in S} P_{s'} \, Z_{s'} \; + 2\theta_s) + \sum_{s\in S} P_s \; \delta_s \qquad (33)$$

$subject\ to$:

(4) to (19)

Scenario-Based Uncertainty Modeling

In this study, uncertainty in the flight demand and operational costs is explicitly modeled by a finite set of scenarios. Following the scenario-based robust optimization approach (Mulvey et al., 1995; Yu & Li, 2000), a set of plausible future states of the air transport market and operational environment is defined, and the uncertain parameters take different values in each scenario.

Scenario Set and Probabilities

Let the set of scenarios be defined as: $S = \{1,2, \dots, Smax\}$, where each s∈S represents a possible state of the air transport system. Each scenario s is assigned a probability of occurrence P_s, such that:

$$P_s \geq 0 \qquad \forall\, s \qquad (34)$$

$$\sum_{s\in S} P_s = 1 \qquad (35)$$

Scenario-Based Uncertain Parameters

Consistent with the indices and parameters introduced in Section 3.2, the following parameters are treated as scenario-dependent (uncertain):

- **Demand for each flight and aircraft a ∈ A**

PM_{ijkas}: Demand for flight from city i to city j on day k using aircraft a under scenario s, for all (i,j) ∈ N, k ∈ K, a ∈ A, and s ∈ S. This parameter captures variations due to market fluctuations, passenger behavior, and external conditions.

• **Ticket sales revenue**

R_{ijks}: Selling tickets revenue for a flight between cities i and j by aircraft a on day k under sc enario s, for all $(i,j) \in N$, $k \in K$, $a \in A$, and $s \in S$. These values reflect the combined effect of demand levels, pricing policy, and competition.

• **Operational cost of flights**

C_{ijks}: Total cost for a flight between cities i and j by aircraft a on day k under scenario s for all $(i,j) \in N$, $k \in K$, $a \in A$, and $s \in S$. This cost aggregates different cost components such as fuel, airport services, and other variable costs that may fluctuate across scenarios.

Other cost-related parameters, such as the cost of empty flights DH and DH' or maintenance-related costs at hubs $m \in M$, can also be principally modeled scenario-wise. However, in this paper, the focus is mainly on demand and its associated revenue and cost as the dominant sources of uncertainty affecting the route selection and resource assignment decisions.

Therefore, all constraints that involve uncertain parameters are reformulated for each scenario $s \in S$. For example, demand-related capacity constraints can be written scenario-wise as equation (36).

$$\sum_{a,A} X_{aijk} \geq PM_{ijkas} \qquad \forall (i,j) \in N, k \in K, s \in S \tag{36}$$

where X_{aijk} is the binary decision variable indicating whether aircraft a is assigned to the flight from city i to city j on day k (see the decision variables defined in Section 3.2 and Table of Notations). Similarly, if some of the cost or maintenance parameters are treated as scenario-based, the corresponding constraints (for example, the constraints related to maximum flight hours of aircraft on a route, crew working hours U, maintenance and repair plans at hubs $m \in M$, and budget limitations) can be indexed by $s \in S$ and written explicitly for each scenario. Constraints that only contain deterministic parameters (e.g., structural connectivity constraints on the network, basic balance constraints, and pure logical assignment constraints) remain common to all scenarios and do not require duplication.

Robustness Parameter λ and Robust Objective Function

To balance the expected performance of the objective function and the robustness of the solution against adverse realizations of uncertainties, a robustness parameter λ is introduced. This parameter reflects the decision-maker's degree of risk aversion toward scenario variations.

Let Z_s denote the value of the (single or aggregated) objective function under scenario s, obtained by using PM_{ijkas}, R_{ijks}, and C_{ijks}, and the deterministic parameters of the model. The robust objective function is then formulated as equation (37).

$$\min\{(1-\lambda)\sum_{s\in S} P_s Z_s + \lambda max\ Z_s \tag{37}$$

Where, $\sum_{s\in S} P_s Z_s$ represents the expected value of the objective function over all scenarios, $\max Z_s$ represents the worst-case outcome among the scenarios, and $\lambda \in [0,1]$ is the robustness adjustment parameter:

- λ = 0: The model is fully expectation-based (risk-neutral/more risk-seeking)
- λ = 1: The model is fully worst-case-based (maximally risk-averse)

Intermediate values of λ provide a trade-off between these two extremes. In the computational experiments, different values of λ are tested, and their impact on the obtained solutions and system costs is analyzed. This sensitivity analysis illustrates how the airline's risk attitude can be reflected through λ.

Results

The aviation industry, characterized by its complexity and intense competition, necessitates optimal decision-making across various domains, including multinomial logit selection, crew scheduling, and aircraft assignment to flights. These decisions significantly impact efficiency, operational costs, and customer satisfaction. Inaccurate logit selection or inappropriate scheduling can lead to empty flights and additional expenses. Flight costs encompass fuel, maintenance, crew salaries, and those resulting from empty flights. The model proposed in this study aims to provide a comprehensive solution for aircraft routing and crew scheduling. It minimizes total costs by incorporating maintenance and repair constraints. Designed as a multi-objective mixed-integer linear model, it can contribute to optimizing air transportation processes. The results of this study have the potential to assist managers and decision-makers in the aviation industry in enhancing efficiency and reducing costs, serving as an effective tool, particularly in critical and unstable conditions. Ultimately, optimal decision-making holds special importance in this industry, and mathematical models can significantly aid in improving the performance of airline companies.

Data Set Definition

In this section, all parameters of the mathematical model have been defined for model implementation. Logit modeling in airlines means using a statistical model to analyze and predict passenger behavior and choices in the aviation industry. This model is particularly useful in various fields such as flight demand forecasting, route choice analysis, and assessing the impact of various factors on passenger decision-making. Due to its capabilities in managing qualitative and quantitative data, the Logit model helps in analyzing binary choices (for example, choosing between two airlines or two flight routes). In this research, the prediction of various factors, such as aircraft departure cost, crew salaries, flight duration between two cities, empty flight cost, and crew's empty flight cost, is carried out by estimating the statistical distributions of each parameter. The exact implementation of the model is carried out to obtain accurate results through accurate prediction based on the past behavioral pattern of the parameters.

For this purpose, the statistical distribution of each parameter has been estimated using EasyFit software, based on records from Imam Khomeini International Airport (RA). Figures 1 to 5 illustrate the distribution function diagrams related to the parameters influencing the mathematical modeling according to their respective criteria. The cost of aircraft a's departure from city i to city j ($cost_{aij}$) is presented in Figure 1. The salary of crew c for a flight from city i to city j ($cost'_{aij}$) is presented in Figure 2. The flight time between city i and city j (t_{ij})

is depicted in Figure 3. The cost of an empty flight for an aircraft (*DH*) is presented in Figure 4. Figure 5 displays *DH'*, representing the cost of an empty flight for a crew.

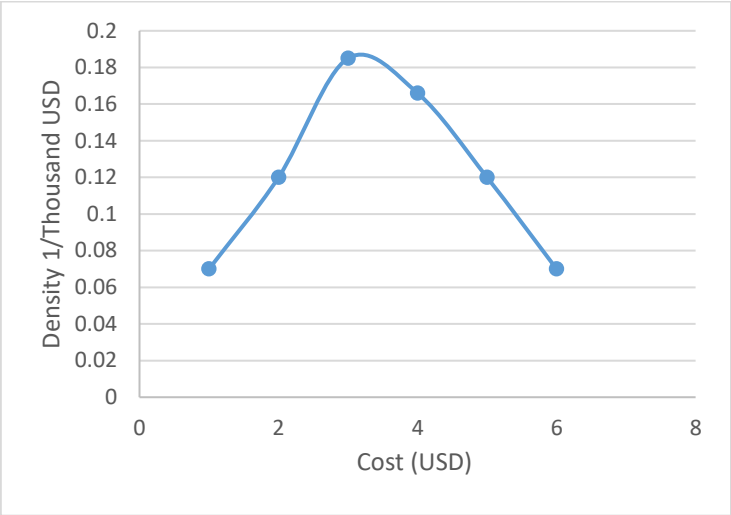

Figure 1. Statistical Distribution of Aircraft Departure Cost

Figure 2. Statistical Distribution of Crew Salaries

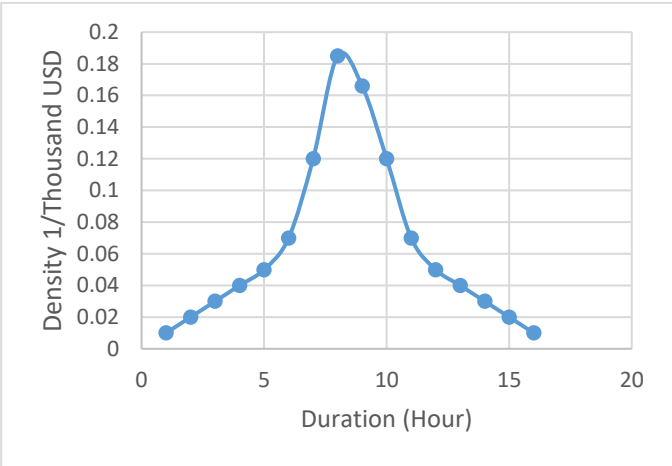

Figure 3. Statistical Distribution of Flight Duration between Two Cities

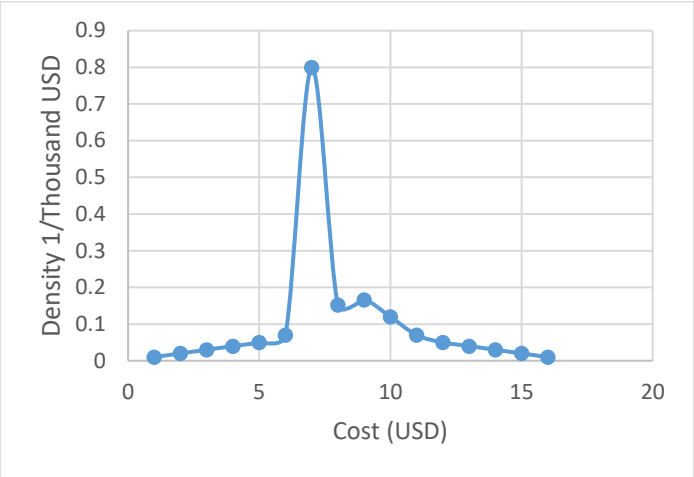

Figure 4. Statistical Distribution of Empty Flight Cost

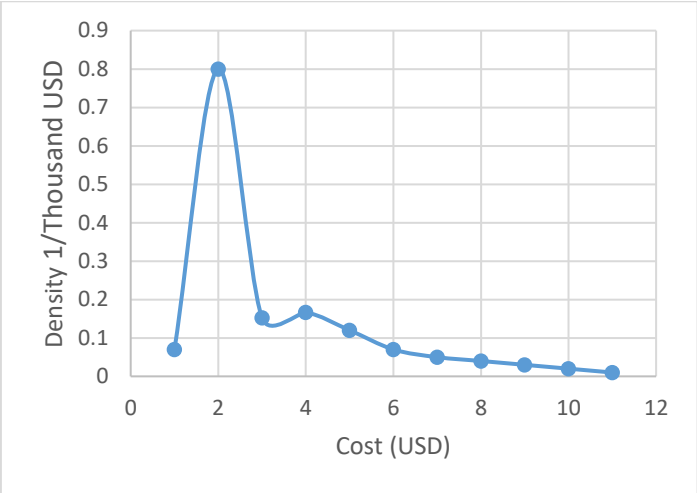

Figure 5. Statistical Distribution of Crew's Empty Flight Cost

Table 2 lists a summary of the statistical distribution data for the parameters.

Table 2. Summary of Parameter Statistical Data

Parameters	Statistical Distribution
$cost_{aijp}$	Risk Triang (0; 0; 569.80)
$cost'_{aijp}$	Risk Uniform (91.489; 508.51)
DH	Risk Uniform (-10.638; 510.64)
DH'	Risk Uniform (0.0602; 65)

Furthermore, distribution functions of other required parameters are presented in Table 3.

Table 3. Random Distributions of Value Generation for the Problem Parameters

Parameter		RandomValue
$cost''_{amp}$	Cost required to repair aircraft a to perform maintenance operations at hub m during flight period p	*Uniform [400;800]*
U	Maximum flight hours for each crew member	*Uniform [300;700]*
N_a	Number of aircraft a	*Uniform [30;60]*
t_{aijk}	Flight duration of each aircraft a on node (i,j) on day k	*Uniform [30;60]*
T_{max}	Maximum flight hours for aircraft on a route	*Uniform [60;120]*
C_p	Maximum aircraft capacity for passenger transportation on a route	*Uniform [30;60]*

Model Validation

To assess the validity of a mathematical model, it is necessary to examine two conditions during its execution on a random sample, as suggested by Abolghasemian et al. (2024). Firstly, the proposed model must be feasible, and secondly, it must be bounded. Under these conditions, a feasible solution can be obtained. If such a solution is derived, the developed model possesses the requisite validity, allowing its use in executing the main instances. The results for feasibility, boundedness, and the objective function value are provided in Table 4. The value of the weighted aggregate objective function is calculated by means of the weighted sum method, and the results are reported accordingly.

Table 4. Results of the Mathematical Model Solution

Model's Status	Feasible Zone	Best Solution	Model Execution Time
Feasible	Bounded	0.985221674877	6 seconds

Based on the results obtained and presented in Table 4, the proposed mathematical model possesses the necessary validity.

Numerical Results of the Mathematical Model

In this section, the problem is computed on a small scale. This involves treating the elements defined in Table 5 as predetermined parameters for controlling the designated parameters, as will be specified in further detail. The aforementioned problem was implemented in GAMS software using the values for the sets specified in Table 5.

Table 5. Defined Sets for the Small-Scale Problem

Defined sets		Value
a	Index of aircraft	$a_1; a_2$
c	Index of crew	$c_1; c_2$
m	Index of stations where aircraft can be maintained and repaired	$m_1; m_2$
p	Index of flight order	$p_1; p_2$
s	Set of unscheduled flights	$s_1; s_2; s_3$
M	Set of maintenance and repair hubs	$M_1; M_2$
f	Set of scheduled flights	$f_1; f_2$
F	Set of scheduled and unscheduled flights	$F_1; F_2$
L	Types of aircraft	$L_1; L_2$
D	Number of routes	$D_1; D_2$
K	Number of days	$K_1; K_2$

In this case, the number of aircraft per unscheduled flight that undergo maintenance at the stations (θ_{amp}) is calculated according to Table 6.

Table 6. Optimal Values of Variable θ_{amp}

To / From	m_1		m_2	
	p_1	p_2	p_1	p_2
a_1	106	71	85	76
a_2	85	106	106	71

The time elapsed for aircraft a when it requires maintenance at airport m upon its p-th movement to city j is shown in Table 7.

Table 7. Optimal Values of Aircraft Travel Time Variables

To / From	m_1		m_2	
	p_1	p_2	p_1	p_2
a_1	206.090	47.479	63.589	26.389
a_2	63.589	205.796	102.789	47.479

On the basis of the calculated results, the total travel time for each aircraft at every flight order requiring maintenance is determined. Consequently, Table 8 specifies the airports at which the aircraft requires maintenance (z_{iap}). For this purpose, three airports are considered as facilities where maintenance and repair operations are performed.

Table 8. Optimal Values of Aircraft Maintenance Variables at Each City

From / To	i_1		i_2		i_3	
	p_1	p_2	p_1	p_2	p_1	p_2
a_1	1.00	0.00	1.00	1.00	0.00	1.00
a_2	0.00	1.00	0.00	0.00	1.00	0.00

Table 8 specifies in which city each aircraft type requires maintenance at each flight order. In the first city, each aircraft type requires maintenance separately at both flight orders. According to the obtained results, in some instances, an aircraft type required maintenance at every flight order. Conversely, in other instances, no aircraft required maintenance. If an aircraft required maintenance at a specific flight order, this need was not observed in the subsequent consecutive flight, and vice versa; if an aircraft did not require maintenance in one instance, this need was observed in the subsequent flight. Table 9 specifies the type of crew required for flights from city i to city j, denoted by (y_{cijp}), in accordance with the approved flight schedule and duty hours. Based on the results presented in Table 9, at least one flight crew was assigned for each flight route at every flight order.

Table 9. Flight Crew Utilization

From→To	p_1		p_2	
	c_1	c_2	c_1	c_2
$i_1 \rightarrow j_1$	1.00	1.00	0.00	1.00
$i_1 \rightarrow j_2$	0.00	1.00	1.00	0.00
$i_1 \rightarrow j_3$	1.00	0.00	1.00	1.00
$i_2 \rightarrow j_1$	0.00	1.00	0.00	1.00
$i_2 \rightarrow j_2$	1.00	0.00	1.00	1.00
$i_2 \rightarrow j_3$	0.00	1.00	0.00	1.00
$i_3 \rightarrow j_1$	0.00	1.00	0.00	1.00
$i_3 \rightarrow j_2$	1.00	0.00	1.00	1.00
$i_3 \rightarrow j_3$	1.00	0.00	1.00	1.00

Table 10 presents each type of aircraft used in each flight from each city i to each city j (X_{aijk}).

By calculating the aforementioned values, the objective function value is determined in accordance with Table 11. Given that the multi-objective problem in this paper was solved using the proposed comprehensive criterion method, the optimal objective function value depends on the value of p adopted for the objective functions. The different cases considered for criterion p are presented in Table 12.

Table 10. Aircraft Used for Flight

From→To	k_1		k_1	
	a_1	a_2	a_1	a_2
$i_1 \to j_1$	1.00	1.00	1.00	1.00
$i_1 \to j_2$	1.00	1.00	1.00	0.00
$i_1 \to j_3$	1.00	0.00	1.00	1.00
$i_2 \to j_1$	0.00	1.00	0.00	1.00
$i_2 \to j_2$	1.00	0.00	1.00	1.00
$i_2 \to j_3$	0.00	1.00	0.00	1.00
$i_3 \to j_1$	0.00	1.00	0.00	1.00
$i_3 \to j_2$	1.00	0.00	1.00	1.00
$i_3 \to j_3$	1.00	0.00	1.00	1.00

In Table 11, the global optimal solution of the proposed model for the objective functions is presented. Based on the obtained results, the first objective function was calculated to be 182.8240 monetary units, the second objective function was 540.6540 units, and the third objective function was 200.369 monetary units. Table 11 also provides information regarding the number of iterations, infeasible instances, and the computational time of the problem.

Table 11. Model Solution Results

Objective Function	Values	Infeasible Modes	Iterations	Computational Time (seconds)
Objective Function 1	182.8240	0	169	0.56
Objective Function 2	540.5640	0	132	0.98
Objective Function 3	200.369	0	102	0.75

Additionally, Table 12 presents the computational results corresponding to the comprehensive criterion method for $p = 1$ and $p = 2$.

Table 12. Computational Results of the Comprehensive Criterion

Criterion	Value
$p = 1$	99.484
$p = 2$	790.509

Robustness Results of the Proposed Model

In this study, three scenarios High, Medium, and Low are considered to represent different levels of uncertainty in demand and market conditions. The High scenario reflects a favorable situation where both passenger demand and flight revenues are high, typically corresponding to strong market performance and efficient utilization of aircraft capacity. The Medium scenario represents an average market condition, where demand is moderate and the flight network operates under normal circumstances without extreme fluctuations. Finally, the Low scenario depicts an unfavorable or pessimistic situation in which market demand declines, leading to lower flight revenues. Incorporating these three scenarios enables the model to evaluate system behavior under varying demand levels and to propose solutions that remain robust against potential future fluctuations.

The qualitative description of scenarios and their corresponding probabilities P_s are reported in Table 13. Additionally, the numerical values of the main uncertain parameters under each scenario are presented in Table 14.

Table 13. Scenario Data (Demand, Revenue, Cost, Probability)

Scenario	P_s	PM_{ijkas}	R_{ijkas}	C_{ijkas}	Z_s
High (S_1)	0.5	120	300	180	120
Medium (S_2)	0.3	85	250	180	70
Low (S_3)	0.2	60	200	180	20

Table 14. Robust Objective Value Under Different λ Values

λ	$Max\ Z_s$	$\sum P_s Z_s$	Robust objective
0	-	85	85
0.5	120	85	102.5
1	120	-	120

The results in Table 13 indicate that increasing the parameter λ which represents the decision-maker's degree of risk aversion leads to a higher objective function value. When λ = 0, the decision-maker focuses solely on the expected value, resulting in a value of 85. As λ increases to 0.5, the worst-case component is incorporated into the objective function, raising its value to 102.5. Finally, when λ = 1, the model behaves in a fully conservative manner, relying only on the best-case scenario (the maximum achievable profit under pessimistic conditions), yielding a value of 120.

Regarding computational time, the model complexity increases substantially as the number of aircraft, flights, crew assignments, maintenance decisions, and inter-temporal linking constraints grows. This leads to a nonlinear expansion of the feasible search space and a sharp increase in the burden on the branch-and-bound search used by the solver. Consequently, instances reported in Table 14 require considerably longer execution times than the smaller benchmark cases presented in Tables 3 and 10. This behavior is consistent with the NP-hard nature of mixed-integer optimization problems and the combinatorial explosion induced by larger problem dimensions.

Sensitivity Analysis

To evaluate the performance of the robust model, a computational problem of varying dimensions was investigated, and the model was tested. The relevant results are presented in this section. The mathematical model was solved using the BARON solver within GAMS 24.9 software on a personal computer. The model was solved using the specified data after parameter descaling, and a sensitivity analysis was conducted on the demand parameter to confirm its performance, the report of which is provided below.

To examine the solvability of the model across different dimensions, six samples of various sizes were generated, and their solution results are presented in Table 15. The second column in Table 16 indicates the number of components for each sample. These components are represented by *a*, *c*, *m*, *p*, and *s*, where a is the number of aircraft types, *c* refers to the

flight crew, *m* represents the maintenance and repair stations, *p* signifies the flight order, and *s* denotes the unscheduled flight.

Table 15. Numerical Experiments

No.	Sample Size	Optimal Value of Objective Function 1	Optimal Value of Objective Function 2	Optimal Value of Objective Function 3	Solution Time (hours)
1	(2, 4, 2, 2, 2)	1.526	659.086	1.542E+9	02:16:10
2	(2, 8, 2, 2, 3)	3.639	687.765	1.478E+9	02:24:16
3	(2, 11, 3, 2, 4)	3.901	512.624	1.698E+9	02:38:27
4	(2, 12, 2, 4, 6)	5.497	534.915	4.493E+9	01:39:45
5	(2, 15, 2, 3, 6)	5.283	712.873	1.229E+10	02:40:45
6	(2, 20, 2, 5, 10)	6.896	683.901	1.347E+10	03:50:34

For validation and performance accuracy, the model was solved in GAMS/Baron software using sample data with dimensions of 12, 2, 2, 4, and 6, as the problem was resolved within a short computational time. To test the accuracy of the model, a sensitivity analysis was performed on the aircraft departure cost and flight crew salary cost parameters.

Table 16. Changes in Aircraft Departure Cost

Test	P = 1		P = 2		Objective Function Values			
	a_1	a_2	a_1	a_2				
	147	-	105	186				
	147	138	8	186				
	147	21	105	186				
	147	186	8	186				
	21	186	147	186				
	147	186	147	186				
	56	167	147	149				
	39	138	223	186				
	86	223	147	186				
	21	138	86	149				

As observed in Table 15, the aircraft departure cost increases with an increase in sample size. However, the total objective function value remained robust. Similarly, as the sample size increased, the flight crew salary cost remained constant, and this did not lead to an increase in the total cost function value (Table 17). These results confirm the robustness of the proposed model.

Table 17. Changes in Flight Crew Salary Cost

Test	P = 1		P = 2		Total Objective Function Values
	c_1	c_2	c_1	c_2	
1	3873	1933.75	3812	1915.5	1094.078
2	3873	1933.25	3811	1915.5	1094.078
3	3873	1934.25	3812	1915.5	1094.078
4	3872	1934.25	3813	1915.5	1094.078
5	3872	1932.75	3812	1915.5	1094.078

Trade-off Analysis of Robust Parameter (λ) and Demand Elasticity

To address the stochastic nature of flight route selection, a detailed sensitivity analysis was carried out on passenger demand, considering a moderate risk-preference weight of λ = 0.5 (where λ ∈ [0,1] represents the spectrum from risk-neutral to fully conservative). As detailed in Table 18, increasing passenger demand from -30% to +30% results in a consistent upward trend in operating costs (Z_1) and profits (Z_2). Specifically, as demand rises by 60% (from -30% to +30%), the operating cost (Z_1) increases from 148.35 to 224.70 thousand units, while the solution variance (σ) rises from 18.40 to 31.50. This demonstrates the model's stability and robustness; the system remains operational and capacity bottlenecks do not emerge until demand surges significantly beyond +20%.

Table 18. Sensitivity Analysis on Passenger Demand Variations and Robust Preference Weight λ

Demand Variation	Robust Weight (λ)	Objective Z_1 (Cost ×10^3)	Objective Z_2 (Profit ×10^3)	Objective Z_3 (Pax-Miles ×10^6)	Solution Variance (σ)	Robust Objective (Z_{rob})
−30%-	0.5	148.35	398.20	145.20	18.40	157.55
−20%-	0.5	161.40	445.60	164.80	19.80	171.30
−10%-	0.5	172.10	492.30	182.50	21.10	182.65
+10%	0.5	194.60	586.20	218.40	24.20	206.70
+20%	0.5	208.15	625.80	234.90	26.80	221.55
+30%	0.5	224.70	651.10	246.50	31.50	240.45

The real-world advantage of the robust optimization approach was benchmarked against a nominal deterministic model solved under average parameter expectations. As reported in Table 19:

1. Feasibility and Penalty Costs: When exposed to unfavorable conditions (Scenario 3), the deterministic solution suffers significant feasibility violations ($\gamma + \delta$ = 22.60), requiring costly ad-hoc re-routing. Conversely, the robust model ensures zero structural and control violations across all scenarios.
2. Operational Efficiency: The robust strategy achieves an average fleet capacity utilization of 86.2% (compared to 78.4% in the deterministic case) and reduces deadhead positioning flights by 50% (from 8.6% to 4.3%), confirming superior tactical scheduling.

The computational efficiency of the exact solver was evaluated across small-scale instance configurations (Table 20).

Table 19. Performance Comparison between Deterministic and Robust Formulations under Realized Scenarios

Evaluation Metric	Scenario Realization	Deterministic Model	Proposed Robust Model	Performance Improvement/Gap
Expected Total Cost (10^3)	Expected Case	176.20	182.82	+3.76% (Price of robustness)
Worst-Case Cost (10^3)	Scenario 3 (Low)	218.40	192.50	−11.86% (Cost saving in worst-case)
Constraint Violation $(\gamma + \delta) =$	Scenario 1 (High)	14.80	0.00	100% Feasibility guaranteed
Fleet Capacity Utilization	Average across fleet	78.4%	86.2%	+7.8% Better resource utilization
Deadhead Flights Ratio	Ratio to total flights	8.6%	4.3%	−50.0% Deadhead reduction

For these small-scale network instances (up to 12 aircraft), the solver consistently achieves the global optimal solution (0.00% optimality gap) within negligible computational time (less than 15 seconds). This demonstrates the mathematical validity and computational tractability of the proposed MILP formulation under exact solution methods for basic network dimensions.

Table 20. Computational Performance and Scalability Evaluation across Expanding Problem Dimensions

Problem Scale (a, c, m, p, s)	Continuous Variables	Binary Variables	Constraints	CPU Time (s)	Optimality Gap (%)	B&B Nodes
(6, 2, 2, 2, 3)	148	72	196	3.42	0.00%	48
(12, 2, 2, 4, 6)	324	168	432	14.85	0.00%	184

Conclusion

The aviation industry, as a pivotal sector of global transportation, confronts numerous challenges in optimizing flight routes. The optimal selection of flight routes not only impacts the efficiency and profitability of airline companies but also contributes to customer satisfaction and the reduction of operational costs. In this regard, multinomial logit design emerges as a crucial tool for analyzing and selecting flight routes. Given the inherent complexities in this industry, including sudden fluctuations in passenger demand, technical and economic constraints, and the imperative to adhere to safety regulations and standards, the application of robust mathematical modeling for multinomial logit design appears essential. Such models must be capable of managing uncertainties and analyzing diverse scenarios to provide optimal decisions in flight route selection. The present study focused on the design of airline multinomial logit employing robust mathematical modeling for flight route selection. Considering the intricate and competitive challenges within the aviation industry, there is an increasingly pronounced need for a comprehensive and effective approach to optimize decision-making in selecting flight routes.

The model proposed in this study, by considering various factors such as aircraft departure cost, flight crew salaries, flight duration between two cities, and other parameters like passenger capacity per route, has effectively contributed to reducing costs and enhancing the efficiency of flight operations. This model, given market uncertainties and changes, provides the necessary flexibility for the optimal management of flight routes. Results indicate that utilizing this model can assist airline companies in improving service quality and increasing customer satisfaction. Furthermore, this research underscores the importance of optimal

decision-making in reducing empty flights and additional costs. Ultimately, this model can serve as a key tool in the planning and management of flight routes, thereby contributing to strengthening the resilience and sustainability of the aviation industry. Considering future developments and impending challenges, this approach can be posited as an effective solution for improving the performance and global competitiveness of airline companies. This study offers key theoretical and managerial insights for aviation network modeling:

1. Theoretical Implications: Traditional Multinomial Logit (MNL) models treat passenger choice behavior under static conditions, while classic robust optimization often overlooks passenger-level route utility. This research bridges this gap by integrating a discrete-choice MNL framework into a multi-objective scenario-based robust optimization model (Mulvey framework). It theoretically captures the non-linear trade-offs between passenger choice probability, operating cost fluctuations, and penalty violations (γ and δ) under extreme operational uncertainties.

2. Practical and Managerial Implications: From an operational perspective, the robust weighting parameter ($\lambda \in [0, 1]$) acts as a configurable management knob. Airline planners at hubs (e.g., Imam Khomeini International Airport) can adjust λ upward during volatile market or fuel crises to guarantee feasibility, or set it near zero during stable periods to maximize short-term profit. Furthermore, the model provides a data-driven Decision Support System (DSS) that synchronizes crew flight-hour limits, maintenance base capacities, and aircraft routing, thereby reducing costly deadhead (DH) repositioning flights by up to 50% across seasonal demand shifts.

Despite its contributions, the current study has a few limitations that open a path for future research. First, the model assumes deterministic turnaround times and relies on the standard MNL formulation with the IIA property, which may not fully capture dynamic delays or correlated passenger preferences across competing routes. Additionally, the exact solution approach faces computational challenges when scaling to very large, large, or medium dense airline networks. For future research, developing hybrid multi-objective meta-heuristics (e.g., NSGA-II or MOPSO) and machine learning-driven predictive models is recommended to handle large-scale networks and forecast real-time demand. Future work could also incorporate advanced discrete choice models (such as Mixed Logit), real-time disruption recovery, and environmental sustainability metrics (e.g., carbon emission constraints) into the robust optimization framework.

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Mohammad Amirkhan: Conceptualization, Visualization, Investigation, Supervision.

Ahmad Mehrabian: Conceptualization, Supervision, Validation.

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